Productive cities: Sorting, selection and agglomeration

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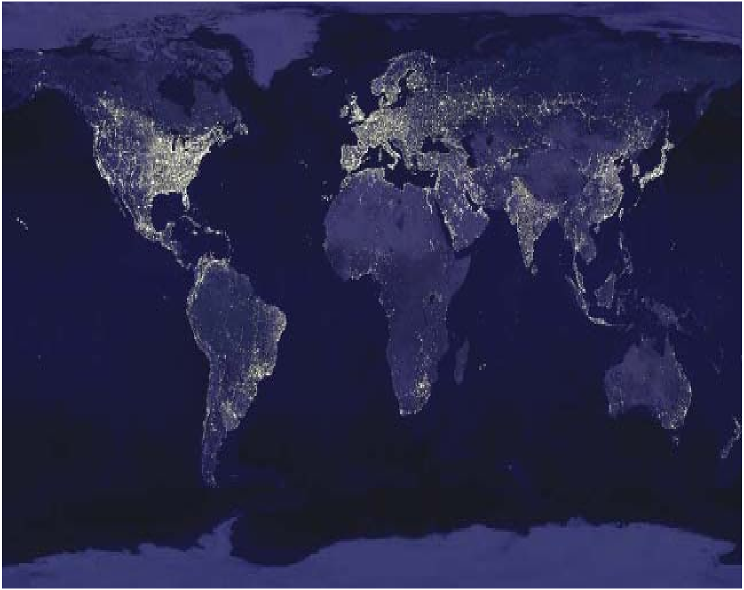
Big cities viewed from space



Motivation I: Spatial inequalities are ubiquitous

Human settlements and production are spatially concentrated

- Cities are the center of economic activity
- e.g. Japan's 3 core metropolitan areas (NOT)
  - cover 5.2% of Japan's land mass
  - host 33% of its population, 31% of its manuf. employment
  - create 40% of its GDP
  - cover .18% of East Asia's area but generate 29% of its GDP!
- Likewise, US's most active counties
  - cover 1.5% of US's land mass
  - represent 41.2% of its manufacturing employment
- Paris metropolitan area (Ile-de-France)
  - Only 12% of available land is used for housing, plants and transportation infrastructure
  - Covers 2.2% of France's area, 19% of its pop., 30% of its GDP
  - Ministère de l'égalité des territoires notwithstanding...

Motivation II: Big cities pay big wages

Urban premium

- Wages and productivity are increasing in city size
- Cities attract the most talented people

Earnings inequalities across cities

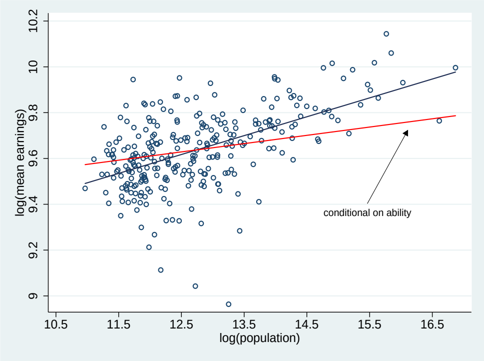


Figure 1. Size–productivity–ability

First aim of our paper (qualitative)

- Provide three possible explanations for urban premium in a unified setting
  - Agglomeration
  - Sorting
  - Selection
  - (We omit natural advantage)

Second aim of our paper (quantitative)

- Provide structural interpretation to these slopes (elasticities)

Motivation III: Cities vary greatly in size

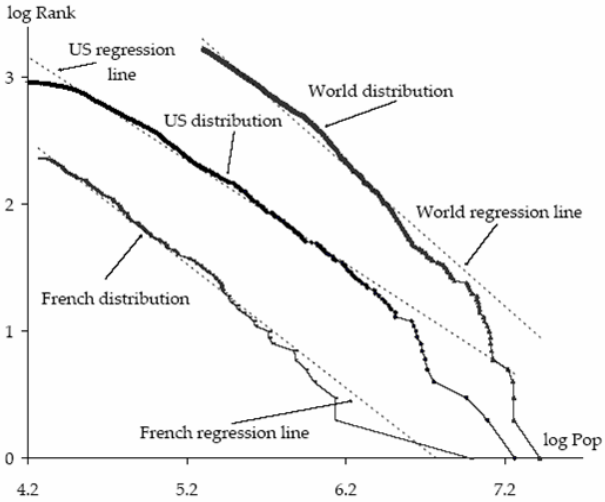
The rank-size rule

- A few large cities, many small towns

$$\ln Rank_c = \ln Size_C + \zeta \ln Size_c,$$

where C is the largest city in the country (Tokyo, Paris, NYC)
  - Zipf's law: $\zeta = -1$

The rank-size rule in the world



Source: Duranton (2008)

Third aim of our paper

- Provide an original explanation for Zipf's law

Plan

- Balancing agglomeration economies and congestion: The Henderson model
- Adding selection and ability sorting across cities: our model
- Equilibrium with talent-homogenous cities
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The Henderson model of cities (American Ec. Rev. 1974)

- Agglomeration economies at the local/urban level
  - Many explanations are plausible, many economic mechanisms have been proposed in the literature (Duranton and Puga 2004)
  - E.g. input sharing as in Ethier's (1982) version of Dixit and Stiglitz (1977)
- Per-capita output in city c

$$\frac{Y_c}{L_c} = A_c L_c^{\varepsilon},$$

where L_c denotes city population and A_c is local TFP

- ε captures agglomeration economies and is related to the mechanism generating local increasing returns
  - e.g. $\varepsilon = 1/(\sigma - 1)$ in the Ethier-Dixit-Stiglitz model

Agglomeration economies in the empirical literature

- Positive association between city size and various measures of productivity (Recall Figure 1 above for the US)
  - Empirically, $\varepsilon \in (0, 0.1)$
- This association is *causal*
  - IV evidence: Ciccone and Hall (1996), Combes, Duranton, Gobillon and Roux (2010)
  - Quasi-experimental evidence: Greenstone, Hornbeck and Moretti (2010)
  - Input-output linkages as a key channel (Holmes 1999; Amiti and Cameron 2007; Ellison, Glaeser and Kerr 2010)

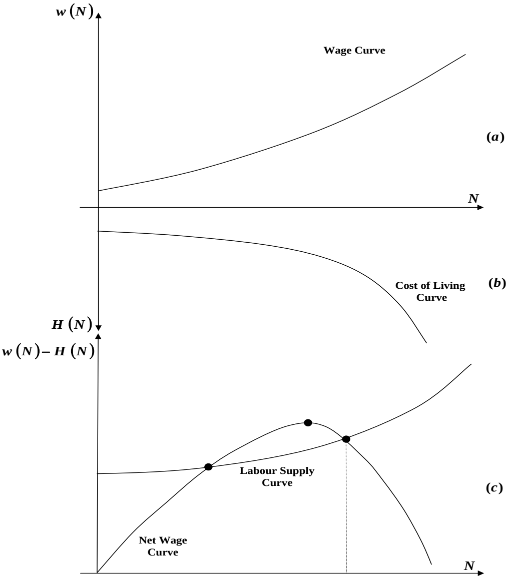
Urban congestion and spatial equilibrium

- Local/urban congestion diseconomies
  - Commuting
  - Competition for the ultimate scarce factor: land
  - Competition for purely local amenities, goods and services
  - Per capita urban costs are proportional to L_c^γ, where γ is the elasticity of the cost of living with respect to city size
- Spatial equilibrium balances the two
  - High wages *compensate* workers for high urban cost of living
  - High worker *productivity* compensates firms for high wages
- Spatial equilibrium with homogeneous agents

$$\omega_c = \omega,$$

for all cities with $L_c > 0$ ($\omega_c < \omega$ otherwise), some $\omega > 0$

Spatial equilibrium in Henderson's model



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Productive cities: Sorting, selection and agglomeration

Objectives

- Build a model of a self-organized urban system with
  - agglomeration economies
  - sorting along ability
  - selection along productivity
- Explain
  - urban premium
  - composition and size distribution of cities
- Model consistent with several stylized facts
  - Provides a static explanation for Zipf's law
  - Allows us to reinterpret extant empirical evidence
  - Little sorting but it matters greatly [a puzzle]
- Central message: $(\gamma - \varepsilon)$ is tiny!

Sorting

- Urban premium increasing with skills (Wheeler, 2001; Glaeser and Maré, 2001)
- Sorting matters (Combes, Duranton, Gobillon, 2008)

Agglomeration

- Size-productivity relationship robust to sorting (CDG, 2008)
- Causal impact of city size on productivity
- IO-linkages are an important source (Holmes, 1999; Ellison, Glaeser, Kerr, 2010)

Selection

- Higher survival productivity cutoff in larger markets (Syverson, 2004)
- But no selection after controlling for agglomeration and sorting (Combes, Duranton, Gobillon, Puga, Roux, 2012)

Model: Timing

1. Talent t of each agent is revealed (c.d.f. G_t)

2. Agents choose a city

3. Luck s of each agent is revealed (c.d.f. G_s)
   $\rightarrow$ Entrepreneurial productivity is $\varphi \equiv t \times s$ (c.d.f. F)
   Worker productivity is φ^a

4. Occupational selection (workers vs entrepreneurs)

5. Market clearing, production, consumption

Model: Preferences and technology

Preferences

- Risk-neutral individuals consume one unit of land and a final consumption good

Technology

- Two-step production process
- Homogenous aggregate output (freely tradable numeraire) in city c

$$Y_c = \left[\int_\Omega x_c(i)^{\frac{1}{1+\varepsilon}} \mathrm{d}i \right]^{1+\varepsilon}$$

produced using local intermediates provided by entrepreneurs

$$x_c(i) = \varphi(i) l_c(i), \quad \text{with} \quad \varphi(i) = t(i) \times s(i)$$

Market outcome

Solving the model backward

- Solve first for prices, quantities and occupations in each city c
- At this stage, individuals take as given:
  - location and own productivity
  - cumulative productivity distribution $F_c(\cdot)$
  - city size L_c
- Individuals self-select into either workers or entrepreneurs
  - We impose $a\varepsilon < 1$
  - i.e. productive agents have a comparative advantage in entrepreneurship

Occupational selection

- Profit maximization yields

$$\pi(\varphi) = \frac{\varepsilon}{1+\varepsilon} Y \left[\frac{\varphi}{\Phi}\right]^{\frac{1}{\varepsilon}} \quad \text{where} \quad \Phi \equiv \left[\int_{\Omega} \varphi(j)^{\frac{1}{\varepsilon}} \mathrm{d}j\right]^{\varepsilon}$$

  - Complementarity between Y and φ
  - Offsetting market crowding or toughness via Φ (aggregate city productivity)
- Agent with productivity φ becomes entrepreneur iff

$$\pi(\varphi) > w\varphi^{a}$$

- yields productivity cutoff for selection into entrepreneurship

$$\underline{\varphi} \equiv \left[\Phi \left(\frac{1+\varepsilon}{\varepsilon} \frac{w}{Y}\right)^{\varepsilon}\right]^{\frac{1}{1-a\,\varepsilon}}$$

City equilibrium

Proposition 1 (existence and selection). Given population, L, and its productivity distribution, $F(\cdot)$, *the equilibrium in a city exists and is unique*.

Proposition 2 (agglomeration). Given $F(\cdot)$, larger cities have higher aggregate productivity, per-capita income, and wages than smaller cities. Productivity cutoff for selection does not depend on city size.

Per-capita city income is

$$\frac{Y}{L} = \left(\int_{\underline{\varphi}}^{+\infty} \varphi^{\frac{1}{\varepsilon}} \mathrm{d}F(\varphi) \right)^{\varepsilon} \left(\int_{0}^{\underline{\varphi}} \varphi^{a} \mathrm{d}F(\varphi) \right) L^{\varepsilon} \,,$$

Urban costs

- Standard monocentric city structure
- Commuting costs $t(x) \propto x^{\gamma}$, where $\gamma > \varepsilon$
- Per-capita urban costs are given by θL^{γ}

Returns to talent are increasing in city size

- Expected utility for individual with talent t

$$\begin{aligned}\mathbb{E}V(t) &= \int_0^{+\infty} \max\{w \times (ts)^a, \pi(ts)\} \mathrm{d}G_s(s) - \theta L^\gamma \\ &= w\, t^a \left[\int_0^{\underline{\varphi}/t} s^a \mathrm{d}G_s(s) + \left(\frac{t}{\underline{\varphi}}\right)^{\frac{1}{\varepsilon}-a} \int_{\underline{\varphi}/t}^{+\infty} s^{\frac{1}{\varepsilon}} \mathrm{d}G_s(s) \right] - \theta L^\gamma\end{aligned}$$

Proposition 3 (complementarity between talent and city population). Conditional of $F(\cdot)$, more talented individuals benefit disproportionately from being located in larger cities:

$$\frac{\partial^2 \mathbb{E}V(t)}{\partial t \partial L} \Big|_{F(.)} \geq 0$$

Location choice

- Location choice to maximize $\mathbb{E}V_c(t)$ for c
- $F(\cdot)$ is endogenously determined (endogenous city composition)
- Distribution of luck identical across all cities
- Assignment problem: matching function $\mu : T \to C$ maps talents into cities $c, c' \in C$:

$$\mu(t) = \left\{ c : \mathbb{E}V_c(t) \geq \mathbb{E}V_{c'}(t), \ \forall c' \in C \right\}.$$

- Self-organized equilibrium: Nobody wants to deviate given the location choices of all other individuals

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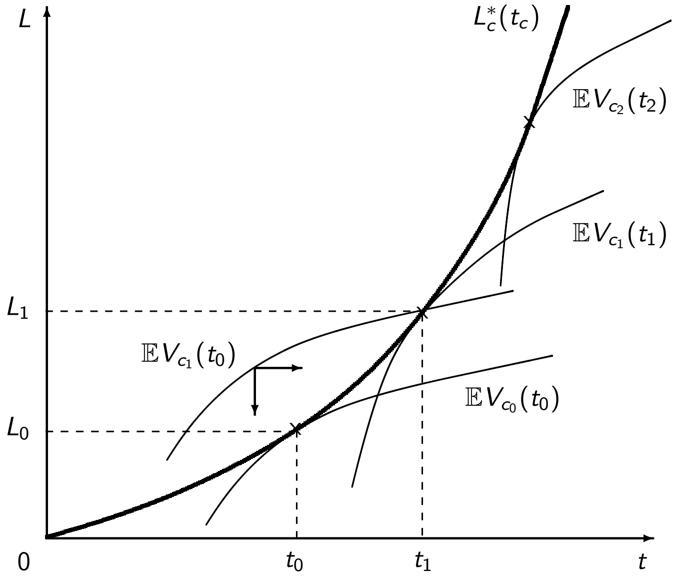
Talent homogenous cities equilibrium

- Symmetric equilibrium unstable if sufficient heterogeneity in talent
- Consider equilibrium with only one type of talent t_c per city (but non-degenerate productivity distribution)
- The productivity cutoff is proportional to talent : $\underline{\varphi}_c = S \times t_c$ for some S and all c (easy to show formally)
  - i.e. sorting induces selection
  - Conditional on sorting, no differences in selection (CDGPR, 2010)

- Assignment problem is tricky since $\mathbb{E}V_c(t)$ not generally supermodular in t and L
- At equilibrium, cities can neither be too small (agglomeration) nor too large (congestion)
- Existence of an equilibrium relationship between talent (productivity) and size



Talent homogeneous equilibrium

Talent homogenous cities equilibrium: Properties

Proposition 4 (Equilibrium population of talent-homogenous cities). Talent-homogeneous cities of *optimal size* are such that:

$$L^o(c) = \left[\xi t(c)^{1+a}\right]^{\frac{1}{\gamma-\varepsilon}}$$

Talent-homogeneous cities of *equilibrium size* are such that

$$L^*(c) = \left[\frac{1+\gamma}{1+\varepsilon}\xi t(c)^{1+a}\right]^{\frac{1}{\gamma-\varepsilon}} \quad \Rightarrow \quad L^*(c) > L^o(c)$$

- Cities are oversized
- If $\gamma - \varepsilon$ is small (as seems empirically the case), then *'mild sorting' goes hand-in-hand with large size differences*

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Talent homogenous cities equilibrium: Zipf's law

Proposition 5 (Number and size distribution of cities). The equilibrium 'number' of cities is proportional to population size Λ and too small relative to the social optimum. The size distribution of talent-homogenous cities converges to **Zipf's law** *regardless of the distribution* of talent t as $\eta \equiv (\gamma - \varepsilon)/(1 + a) \to 0$.

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Quantitative applications and quantitative implications

We use the model to provide structural interpretation of extant regressions and conduct welfare analysis

Estimating γ and ε

- γ is the regression coefficient of $\ln w_c = \alpha^I + \gamma \ln L_c + \epsilon_c^I$
- ε is the regression coefficient of $\ln w_c = \alpha^{II} + \varepsilon \ln L_c + t_c + \epsilon_c^{II}$
- Using US data we find $\widehat{\gamma} = .082$ and $\widehat{\varepsilon} = .051$
- Remarkably, γ is also the regression coefficient of $\ln t_c = \alpha^{III} + \gamma \ln L_c + \epsilon_c^{III}$
- For this regression we find $\widehat{\gamma} = .068$

Cities are *naturally* oversized

- by a huge factor (close to Euler's e)
- but welfare cost of oversize is negligible
- Why? Because γ, ε and $(\gamma - \varepsilon)$ are tiny.

Revisiting the findings of Gennaioli, La Porta, Lopez-de-Silanes, and Shleifer (2012)

- Macro data: 1499 regions of 105 countries
- Regressing output per capita y_c on region size and stuff

$$\begin{aligned} \ln(y_c) &= \varepsilon \ln L_c + f\left(G_{t,c}(\cdot), G_s(\cdot)\right) \\ &\approx 0.068 \log L_c + 0.257\,\text{Educ}_c + \text{controls}_c + \upsilon_c \end{aligned}$$

Revisiting the findings of Gennaioli *et al.* (cont.)

- Extend our model for limited span-of-control (Lucas 1978)
- Micro data: 6314 firms in 76 regions of 20 countries
- Regressing firm revenue Z_i on education

$$\ln Z_i = 0.126 \log L_{c(i)} + 0.073\,\mathrm{Educ}_{c(i)} + 0.860\,N_i + 0.017\,\mathrm{Educ}_i^W + 0.026\mathrm{Educ}_i^E + \mathrm{controls}_{c(i)} + \upsilon_i$$

- This implies extremely high returns to education for entrepreneurs in the framework of Gennaioli *et al.* (26%)

Our interpretation:

- Self-selection of the most talented into entrepreneurship: coefficient on $Educ_i^E$ is biased upwards
- Self-selection of the least talented into workers: coefficient on $Educ_i^W$ is biased downwards

A novel interpretation of the finding of Davis and Ortalo-Magné (2011)

- Housing expenditure shares may be constant without imposing Cobb-Douglas preferences

Summary

- **Agglomeration, selection and sorting** interact to explain the urban premium, the composition, and the size distribution of cities
- Model captures key stylised facts, useful for reinterpreting empirical evidence in a unified framework
- Provides a static explanation for **Zipf's law** in Henderson-like model
- Provides an explanation for the **sorting puzzle**