Microeconomics: Adverse selection, screening and the revelation principle

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Abstract

This note introduces the important concepts of price discrimination, incentive- and rational-compatibility constraints, and revelation principle in the context of screening types against adverse selection.

1 Adverse selection

Consider the following game as an example:

- Players: a monopolist and many different buyers.
- Monopolist produces and sells a homogenous good.
- Buyers know their valuation of the good; the monopolist does not. Their valuation is low with probability p_L and high with probability p_H, with $p_L + p_H \equiv 1$.
- Seller's payoff: $\pi(q, T) = T - cq$, where c is the constant marginal cost of production.
- Buyer's payoff: $U(q, T; \theta) = \theta v(q) - T$, where $\theta \in \{\theta_L, \theta_H\}$, with $\theta_L < \theta_H$ and $v(0) = 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$.

1.1 Benchmark: no asymmetric information

In this case the monopolist can price discriminate (i.e. set different tariffs for different types). Thus the seller sets $\max_{T_j, q_j} \pi(q_j, T_j)$ such that $U(q_j, T_j; \theta_j) \geq 0$, $j = H, L$. [interpret this *participation constraint*]. This is equivalent to

$$\max_{q_j} \theta_j v(q_j) - cq_j.$$

The FOC to this problem is

$$\theta_j v'\left(q_j^0\right) = c$$

so that $q_L^0 < q_H^0$ by the SOC [make sure you understand the economics of this]. Also,

$$T_j^0 = \theta_j v\left(q_j^0\right).$$

This is known as *perfect price discrimination*: this is an efficient allocation, though it leaves consumers with no surplus. This is an important result.

1.2 Asymmetric information and the revelation principle

Consider now the situation in which only the buyer knows her type. The seller aims to design two contracts, one for each type. The revelation principle states that we need only consider to points on the contract curve: (q_L, T_L) and (q_H, T_H). Can the contracts above be part of an equilibrium in this case? In other words, are such contracts part of a PBE? In yet other words, has any type any incentive to cheat and pretend to be of the other type?

Consider a deviation from buyers with a high type:

$$\begin{aligned} U\left(q_L^0, T_L^0, \theta_H\right) &= \theta_H v\left(q_L^0\right) - T_L \\ &= \theta_H v\left(q_L^0\right) - \theta_L v\left(q_L^0\right) \\ &= \left(\theta_H - \theta_L\right) v\left(q_L^0\right) \\ &> 0 \end{aligned}$$

so the high type has a unilateral incentive to pretend to be a low type. It immediately follows that the contracts (q_L^0, T_L^0) and (q_H^0, T_H^0) cannot be part of a PBE because they are not *incentive compatible*.

2 Incentive compatible contracts (perfect baysian equilibrium)

Using the revelation principle, the seller's problem is

$$\max_{T_L, T_H, q_L, q_H} p_L\left(T_L - cq_L\right) + p_H\left(T_H - cq_H\right)$$

such that

$$\theta_H v\left(q_H\right) - T_H \geq \theta_H v\left(q_L\right) - T_L \quad \text{(ICH)}$$

$$\theta_L v\left(q_L\right) - T_L \geq \theta_L v\left(q_H\right) - T_H \quad \text{(ICL)}$$

(these are the incentive compatibility constraints of types H and L, respectively) and

$$\theta_H v(q_H) - T_H \geq 0 \tag{IRH}$$
$$\theta_L v(q_L) - T_L \geq 0 \tag{IRL}$$

(these are the *individual rationality*, or participation constraints, of types H and L, respectively).

Solving this problem is a daunting task. However we may simplify it as follows.

First, note that ICH and IRL together yield IRH, so the latter is redundant; indeed:

$$\begin{aligned} \theta_H v(q_H) - T_H &\geq \theta_H v(q_L) - T_L \\ &> \theta_L v(q_L) - T_L \\ &\geq 0. \end{aligned}$$

Second, we have established above that the high type has an incentive to cheat. So we will solve the problem assuming that the low type does not cheat and we will check that this is true only later. Thus we are left with the following problem:

$$\max_{T_L, T_H, q_L, q_H} p_L (T_L - cq_L) + p_H (T_H - cq_H)$$

s.t.

$$\begin{aligned} \theta_H v(q_H) - T_H &\geq \theta_H v(q_L) - T_L \\ \theta_L v(q_L) - T_L &\geq 0. \end{aligned}$$

Third, we note that ICH and ICL will bind at optimum [can you see why?]. Thus, the problem above is equivalent to

$$\max_{q_L, q_H} p_L [\theta_L v(q_L) - cq_L] + p_H [-(\theta_H - \theta_L) v(q_L) + \theta_H v(q_H) - cq_H].$$

Let $p_H \equiv 1 - p$ and $p_L \equiv p$; the FOC to this problem are

$$\begin{aligned} \theta_H v'(q_H^*) &= c \\ \theta_L v'(q_L^*) \left[1 - \frac{1-p}{p} \frac{\theta_H - \theta_L}{\theta_L}\right] &= c \end{aligned}$$

if

$$\frac{1-p}{p} \frac{\theta_H - \theta_L}{\theta_L} < 1$$

and $q_L^* = 0$ otherwise, and

$$\begin{aligned} T_L^* &= \theta_L v(q_L^*) \\ T_H^* &= \theta_H v(q_H^*) - (\theta_H - \theta_L) v(q_L^*), \end{aligned}$$

that is, the two-part tariff is efficient for the high type (i.e. $q_H^* = q_H^0$) but not for the low type ($q_L^* < q_L^0$): this is to avoid H pretending to be of a low type and buying the contract designed for L. Note that the high-type retains some *informational rents* as a result, i.e.

$$U\left(q_H^*, T_H^*, \theta_H\right) = \left(\theta_H - \theta_L\right) v\left(q_L^*\right) > 0,$$

but the low type does not: $U\left(q_L^*, T_L^*, \theta_L\right) = 0$.

Check that ICL holds at the PBE contracts.