Household sorting in the city

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What we do

- Introduce (two-dimensional) household heterogeneity in an otherwise standard monocentric city model
- Study the spatial allocation of households in this context
- Introduce a notion of *spatial equilibrium* that is consistent with household heterogeneity
- Revisit central results in the field in this environment
- Solve the problem as an assignment problem

What we find

- The Alonso-Muth (AM) condition holds *but it is idiosyncratic* and local
- Relationship among urban aggregates breaks down and it *depends on the household composition of the city*
- *The Henry George Theorem fails*: household heterogeneity and sorting create individual rents that cannot be taxed

Motivation

- Household heterogeneity is a central feature of the world
- Household income sorting is imperfect
- Most extent models of urban land use assume a discrete number of household types (typically one)
- ... and/or feature perfect sorting, which is counterfactual

Our contribution in context I: spatial equilibrium concept

- AMM spatial equilibrium: utility is equalized across locations [households are homogenous]
- Fujita (1989) spatial equilibrium: the mass of indifferent agents is **one** [discrete types of households]
- Beckman (1970), Brueckner Thisse Zenou (2002) and Brueckner and Selod (2006): household heterogeneity is one-dimensional [income], which leads to perfect sorting
- We add one level of household heterogeneity [income + preference for leisure time]
- Our spatial equilibrium: the mass of indifferent agents is **zero**

Our contribution in context II: HGT and urban aggregates

- Arnott and Stiglitz (1979): Henry George Theorem (HGT) holds under pretty general conditions
- The key condition we relax: households are heterogenous *and* they sort across residential locations
- Arnott and Stiglitz (1981): simple relationship between aggregate land rent and aggregate commuting cost
- Household heterogeneity-cum-mobility creates rents that are not capitalized in land prices

The HGT and the simple relationship among urban aggregates break down in our context

Start with a simple model

Simple model setup

- N individuals in the city, denoted by $n \in [0, N]$
- Preferences:

$$U\left(x; w(n), c(n)\right) = w(n) - c(n)T(x) - R(x)$$

- Each endowed with a wage $w(n) > 0$ earned at CBD and an idiosyncratic commuting cost $c(n)T(x)$
- $R(x)$ is the equilibrium bid-rent curve

Simple model setup (cont.)

- Preferences:

$$U(x; w(n), c(n)) = w(n) - c(n)T(x) - R(x)$$

- Idiosyncratic commuting cost $c(n)T(x)$:
  - location x is characterized by its distance from CBD
  - $c(n)T(x)$ is n's commuting cost when residing in x
  - $T(x) > 0$ is common component with $T'(x) > 0$
  - $c(n) > 0$ is idiosyncratic distaste for commuting
  - convenient reduced-form for earnings heterogeneity – for instance:

$$c(n) = w(n) \times \varepsilon(n) \quad \text{with} \quad \varepsilon(n) \perp w(n)$$

  - w.l.o.g. rank individuals so that $c'(n) < 0$
- $F(T, w)$ is the joint distribution

Spatial equilibrium I: problem and FOC

- Each household solves

$$\max_{x} U(x;n) = w(n) - c(n)T(x) - R(x)$$

taking $R(x)$ as given

Spatial equilibrium I: problem and FOC (cont.)

- Assuming T and c are continuously differentiable yields the Alonso-Muth (AM) condition:

$$-R'(x) = c(n)T'(x)$$

  i.e. the marginal cost of residing closer to the CBD (common to all) is equal to the marginal benefit (idiosyncratic)
- Thus bid-rent curve depends on equilibrium assignment of households to location, $n(x)$

Spatial equilibrium II: assignment

- Necessary condition for maximum: $-R''(x) < 0$

Assortative Matching Equilibrium (AME)

- Tentative assignment of households to location, $\alpha : x \mapsto n$,

$$n = \alpha(x) = x$$

- Together with the FOC, $-R'(x) = c(n)T'(x)$, this yields

$$R'(x) = -c(x)T'(x),$$

  which pins down the the slope of the bid rent curve.
- Assuming a linear monocentric city on a coastline with

$$R(N) = R_A,$$

  which pins down the intercept of the bid rent curve.

- Integrating the bid rent curve:

$$R(x) = R_A + \int_x^N c(y)\mathrm{d}T(y)$$

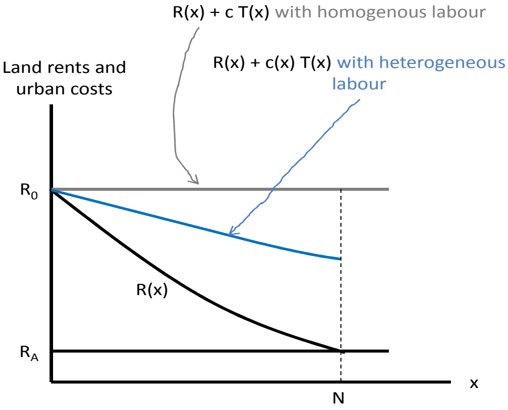
so that

$$R_0 \equiv R(0) = R_A + \int_0^N c(y)\mathrm{d}T(y)$$

- Idiosyncratic urban costs:

$$\begin{aligned} R(x) + c(x)T(x) &= R_0 - \int_0^x [c(y) - c(x)]\mathrm{d}T(y) \\ &\leq R_0 \end{aligned}$$

with strict inequality for all $x > 0$

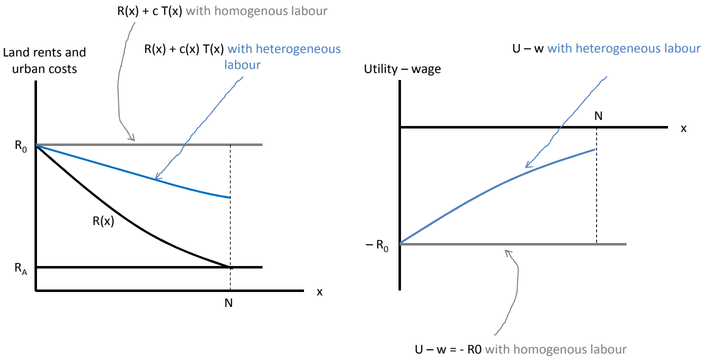


R(x) + c T(x) with homogenous labour
R(x) + c(x) T(x) with heterogeneous labour
Land rents and urban costs
R0
R(x)
RA
x
N

- Equilibrium individual welfare

$$\begin{aligned} V(x;(w(x),t(x)) &= w(x) - R_0 + \int_0^x [c(y) - c(x)] \mathrm{d}T(y) \\ &= w(x) - R_A - c(x)T(N) \\ &+ \int_x^N [c(x) - c(y)] \mathrm{d}T(y) \end{aligned}$$

- Individual equilibrium utility depends city composition
- Heterogeneity creates individual *rents* (standard result in assignment literature)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Land rents and urban costs
R0
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x
N
U – w with heterogeneous labour
Utility – wage
N
x
– R0
U – w = - R0 with homogenous labour

- Urban aggregates at the equilibrium assignment:

$$
\begin{aligned}
\int_0^N [R(x) + c(x)T(x)]\mathrm{d}x &= NR_0 \\
&+ \int_0^N \int_0^x [c(x) - c(y)]\mathrm{d}T(y)\mathrm{d}x \\
&= NR_0 + \int_0^N \int_0^x T(y)\mathrm{d}c(y) \\
&< NR_0
\end{aligned}
$$

by $c' < 0$.

- Final inequality holds because of household heterogeneity

Spatial equilibrium III: imperfect income sorting

Assortative Matching Equilibrium (AME) exists and it is unique

- Perfect household sorting along dimension c (idiosyncratic commuting costs)
- Households of various wages mix unless

$$\mathrm{Corr}\{c(n), w(n)\} \in \{-1, 1\}$$

- If

$$\mathrm{Corr}\{c(n), w(n)\} > 0$$

  then earnings distribution close to CBD stochastically dominates the earnings distribution in the suburbs
  - For instance: $c(n) = w(n)\varepsilon(n)$, with $\varepsilon(n) \perp w(n)$

Equilibrium properties of this simple model

- AME exists and it is unique
- Households of various wages mix in general
- Alonso-Muth (AM) condition holds: $R'(x) = -c(x)T'(x)$

Household heterogeneity creates individual rents. As a result:

- Proportionality between CBD rent R_0 and population size N breaks down
- Henry George Theorem breaks down

Allowing for endogenous housing consumption

Model setup

- N individuals in the city, denoted by $n \in [0, N]$
- Preferences:

$$U(x, h; w(n), c(n)) = u(h) + w(n) - c(n)T(x) - R(x)h$$

  where

  - h is endogenous consumption of housing/land
  - u is increasing and concave

Spatial equilibrium and housing consumption

- Each household solves

$$\max_{x,h} U\left(x, h; n\right) = u(h) + w(n) - c(n)T(x) - R(x)h$$

taking $R(x)$ as given

Spatial equilibrium and housing consumption

- Optimal housing consumption:

$$u'(h) = R(x)$$

  thus optimal h is decreasing in land price.

- Assuming T and c are continuously differentiable yields the AM condition:

$$-R'(x)h = c(n)T'(x)$$

  Thus bid-rent curve depends on equilibrium assignment of households to location, $n(x)$

Spatial equilibrium and housing consumption (cont.)

- The associated second order conditions are

$$u''(h(n,x)) < 0,$$

which holds by assumption,

$$-c(n)T''(x) - R''(x)h(n,x) < 0,$$

and

$$u''(h(n,x))[c(n)T''(x) + R''(x)h(n,x)] > [R''(x)]^2,$$

which **holds only if** $T'' < 0$.

- This necessary condition is consistent with empirical evidence in Combes, Duranton and Gobillon (2013)

Spatial equilibrium II: assignment

Can Assortative Matching be an equilibrium?

- Tentative assignment of households to location:

$$n = \alpha(x) \quad \text{with} \quad \alpha'(x) > 0$$

- Together with the FOCs, this yields

$$R(x) = u'(h(x)), \tag{1}$$

  where $h(x) \equiv h(\alpha(x), x)$, and

$$R'(x)h(x) = -c(x)T'(x) < 0, \tag{2}$$

  which implies $R' < 0$ by $T' > 0$.

- Two differential equations in R and h.
- Assuming a linear monocentric city on a coastline pins down the intercept of the bid rent curve: $R(L) = R_A$.

Spatial equilibrium II: assignment (cont.)

- Housing consumption is equivalent to land consumption:

$$\mathrm{d}x = h(x)\mathrm{d}n$$

  Integrating yields

$$x = \alpha^{-1}(n) = \int_0^n h(\nu)\,\mathrm{d}\nu,$$

  where $h(n) \equiv h\left(n, \alpha^{-1}(n)\right)$.

- Land consumption of the monocentric city is thus equal to

$$\begin{aligned} L &= \int_0^N h(n)\,\mathrm{d}n \\ &= \int_0^N c(n)\frac{T'\left(\alpha^{-1}(n)\right)}{-R'\left(\alpha^{-1}(n)\right)}\mathrm{d}n. \end{aligned}$$

Spatial equilibrium III: equilibrium bid rent curve

- Merging the two differential equations (1) and (2) yields:

$$c(x)T'(x) + u''(x)h'(x)\alpha'(x) = 0 \tag{3}$$

  Thus $h' > 0$ at AME by $\alpha' > 0$, $T' > 0$, and $u'' < 0$.

- Bid rent curve

$$R(x) = R_A + \int_x^L c(y)\frac{T'(y)}{h(y)}\mathrm{d}y,$$

  where the function h is the solution to (3).

- The system is bloc recursive.

Properties of the Assortative Matching Equilibrium (AME)

- For a wide class of preferences, this *assortative matching equilibrium* (AME) exists and it is unique
- Mixed neighbourhoods: in general, household with heterogenous earnings mix, albeit imperfectly
- Density decreases as distance from CBD increases (pure substitution effect in this model)
- Alonso-Muth (AM) condition holds

Household heterogeneity creates individual rents. As a result:

- Proportionality between CBD rent R_0 and population size N breaks down
- Henry George Theorem (HGT) breaks down

Note: multiple equilibria may arise.

Concluding remarks

Summary and conclusions

- Design a simple model without income effect on housing demand (reduced form)
- Yet able to generate a density schedule that is decreasing in distance from CBD
- Households of various incomes mix in heterogeneous neighbourhoods
- Alonso-Muth (AM) condition holds locally
- Simple relationships among urban aggregate and HGT break down
- Household heterogeneity creates individual rents that are not capitalised in land prices and that cannot be taxed