Guide to Calculations by Footnote: "Trade and growth with heterogeneous firms"

(not for publication, will be posted on the web)

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4. The constant operating profit mark-up follows directly from the standard Dixit-Stiglitz result of constant cost-price mark-up; for example, rearranging the first order condition for local sales, $p\left(1-1/\sigma\right)=a$ we get that operating profit earned on local sales is $(p-a)c=pc/\sigma$, where c is local sales.

7. The direct approach to formulating the condition for zero-expected-profit-from-innovation is to calculate the expected benefit net of market entry costs. Starting from the well-known CES demand function, the expected operating profit of a firm that draws its '*a*' from the distribution G[*a*] will be:

$$\int_0^{a_D}\{\frac{a^{1-\sigma}(E/\sigma\gamma)}{\int_0^{a_D}a^{1-\sigma}ndG[a|a_D]+\phi\int_0^{a_X}a^{1-\sigma}ndG[a|a_D]}-F_D\}dG[a]$$
$$+\int_0^{a_X}\{\phi\frac{a^{1-\sigma}(E/\sigma\gamma)}{\int_0^{a_D}a^{1-\sigma}dG[a|a_D]+\phi\int_0^{a_X}a^{1-\sigma}dG[a|a_D]}-F_X\}dG[a]$$

since $ndG[a|a_D]$ gives the frequency of *a*'s among varieties that are actually produced. Using our notation $\overline{m}^{1-\sigma}$, this becomes: $\int_0^{a_D}\{a^{1-\sigma}\frac{E}{\sigma\overline{m}^{1-\sigma}\gamma n}-F_D\}dG[a]+\int_0^{a_X}\{\phi a^{1-\sigma}\frac{E}{\sigma\overline{m}^{1-\sigma}\gamma n}-F_X\}dG[a]$

which simplifies to $(E/\sigma\overline{m}^{1-\sigma}\gamma n)(A[a_D]+\phi A[a_X])$ minus $(G[a_D]F_D+G[a_X]F_X)$, where $A[x]\equiv\int_0^x a^{1-\sigma}dG[a]$. Notice, however, that (3) implies $G[a_D]=(A[a_D]+\phi A[a_X])/\overline{m}^{1-\sigma}$, since $G[a|a_D]=G[a]/G[a_D]$. Thus the benefit less market-entry costs is $G[a_D](E/n\sigma\gamma)-G[a_D]F_D-G[a_X]F_X$. This is set equal to F_I in the direct approach. Dividing this through by $G[a_D]$ validates our indirect approach in (6).

We adopt the indirect approach since it allows us to deal more clearly with growth and corresponds more closely to Tobin's insightful approach to characterising investment in a general equilibrium setting. It also allows a direct comparison with standard endogenous growth models (which do not have market-entry costs) and it facilitates analysis of growth effects by concentrating the impact of openness and parameter changes in the expected cost of getting a winner.

8. See Baldwin and Forslid (2000), Proposition 1 for a general proof that the direct impact on Tobin's q is a sufficient statistic for growth effects in this sort of model. Intuitively, this is obvious since anything that raise Tobin's q encourages more investment and this, in an endogenous growth model, results in faster growth. The faster growth, in turn, returns Tobin's q back to its steady state value of unity. All the variables in the numerator of Tobin's q are either endogenous (e.g. E) or are parameters unrelated to trade liberalisation.

9 and 10. Here we work out the whole dynamic system. In the course of the discussion, the naturalness of using L_I as the state variable becomes clear, but intuitively, the key to constant equilibrium growth is that a time-invariant amount of the primary factor, labour, is devoted to knowledge creation and this knowledge creation process is not subject to diminishing returns with respect to the amount of knowledge created. Since the amount of labour involved in knowledge creation is the key, it is natural to take it as the variable that stops evolving in steady state, i.e. as the state variable.

Formally, income is L+rW, where W is wealth, r is the rate of return, and $\dot{W}$=L+rW-E describes wealth accumulation. All variables are measured in terms of the numeraire, i.e. labour. The Hamiltonian is $e^{-\rho t}\ln(E/P)+\omega(L+rW-E)$ and r and the path of P is exogenous to the consumer/saver. The necessary conditions are $e^{-\rho t}/E=\omega$ and $-\omega r=\dot{\omega}$ plus a transversality condition. Manipulation involving the time derivative of the first condition and substitution of the second condition yields the Euler equation.

The two differential equations are:

$$\dot{E}/E = r - \rho, \qquad \frac{\dot{n}}{n} = \frac{L_I}{p_K \bar{\kappa}}$$

Doing a standard change-of-variables transformation, we take L_I and $g \equiv \dot{n}/n$ as the state variables and, using (10), the system becomes:

$$\frac{-\dot{L}_I}{L - L_I} = r - \rho, \qquad \frac{\dot{n}}{n} = \frac{L_I}{p_K \bar{\kappa}}$$

As usual, changing state variables has no impact on the system dynamics. As is well know from decades of growth models, this system is saddle path stable. Also, as in most endogenous growth models, there is no transitional dynamics since the saddle path is a point, namely the steady state equilibrium. The system jumps immediately to the steady state since otherwise the system would violate the transversality conditions. All these assertions are proved at great length and generality in Grossman and Helpman (1991) and other textbooks on growth. The exact proof in this case can be found in Baldwin and Forslid (2000) which uses Tobin's q approach to evaluate the growth effects of trade in a homogenous goods model.

10. Our use of 'state variable' is somewhat unconventional in the standard terminology of economic dynamics (where state variable means a non-jumper and co-state and control variables are jumpers). Mathematically speaking, the state vector is the set of variables that defines the state of the system fully. This includes variables that can jump or only move smoothly. Thus what economists usually call control variables, co-state variable and state variables are, mathematically speaking, all state variables. After all, one needs to know them all in order to fully characterise the state of the system. For example, in the simple differential equation $\dot{x} = \rho x$ x is the state variable whether it is a jumper or not. The trick of taking L as numeraire and L_I as a state variable opens the door to the intuitive simplification of working with the static economy representation of the dynamic path, i.e. focusing

on the share of primary resources devoted to creation of new 'capital' and the creation of consumption goods. Anything that raises the share of L devoted to knowledge creation is pro-growth.

11. The only asset in this model is the ownership of a variety. The income stream, measured in terms of the numeraire, corresponds to the flow of operating profits. For example, the present value of a D-type's operating profit is discounted back to t=0 at the rate of pure time preference and the stream of operating profit is falling at the rate that n rises, since higher n means lower sales and thus lower operating profit for an existing variety. In symbols, $\int_o^\infty e^{-\rho t}\pi[a,n(t)]dt$ where $\pi[a,n(t)]$ is the function that relates the firm's 'a' and the time-varying n to the level of operating profit. Using (3), this becomes: $\int_o^\infty \left\{ e^{-(\rho+g)t}(\frac{a}{\overline{m}})^{1-\sigma}\frac{E}{\sigma n_0}\right\}dt$. Noting that $\overline{m}$ is time-invariant, and $n(t)=n_0e^{gt}$ on the steady state growth path (n_0 is the initial n), we solve the integral to get: $(a/\overline{m})^{1-\sigma}E/\sigma n_0\gamma$, where $\gamma=\rho+g$. In other words, discounting at the rate of ρ an income stream that falls at a rate of g, is like discounting a constant income stream at the rate of ρ+g.