Oligopoly and cartel: the Cournot case.

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Abstract

In this note I develop the Cournot cartel and oligopoly with linear demand and cost functions and homogeneous good.

1 Oligopoly

The set up is as follows:

- Players: firms $i = 1, ..., N$.
- Firms chose quantities: $q_i \geq 0$.
- Demand: $p = A - bQ$, where $Q \equiv \sum_{i=1}^{N} q_i$. Let

$$q_{-i} \equiv Q - q_i. \tag{1}$$

- Profits:

$$\pi(q_i; q_{-i}) = (p - c) q_i = (a - bq_i - bq_{-i}) q_i, \tag{2}$$

where $a \equiv A - c$.

1.1 Best response and the monopoly case

Firm i maximises profits taking q_{-i} as given; in mathematical symbols: $\max_{q_i} \pi_i(q_i; q_{-i})$. The first order conditions are, for all i,

$$0 = \frac{\partial \pi(\cdot)}{\partial q_i} \equiv a - bq_{-i} - 2bq_i \tag{3}$$

so that the best-response function is

$$q_i = BRF_i(q_{-i}) \equiv \frac{a - bq_{-i}}{2b}.$$

From this expression, we can immediatly infer the monopoly quantity,

$$Q^M \equiv BRF_i(0) = \frac{a}{2b}$$

and, using (2), the monopoly profit:

$$\pi^M = \pi(Q_M; 0) = \left(\frac{a}{2}\right)^2 \frac{1}{b}. \tag{4}$$

1.2 Nash equilibrium

At a Nash equilibrium in quantities, denoted by the vector q^{NE}, $q_i^{NE} = BRF_i\left(q_{-i}^{NE}\right)$ for all i [you should be able to interpret this way of defining a Nash equilibrium by now]. Using (3) and (1) and summing over all i's yields

$$0 = aN - b(N-1)Q^{NE} - 2bQ^{NE}$$

so that

$$Q^{NE} = \frac{N}{N+1}\frac{a}{b}$$

and

$$q^{NE} = \frac{1}{N+1}\frac{a}{b}.$$

In turn, using (2), we obtain

$$\pi^{NE} \equiv \pi\left(q_i^{NE}; q_{-i}^{NE}\right) = \left(\frac{a}{N+1}\right)^2 \frac{1}{b}, \tag{5}$$

which is smaller than π^M by inspection of (4) and (5) (recall that $N \geq 1$).

2 Cartel

The set up is as follows:

- Time is discrete and runs indeffinitely: $t = 0, 1, 2, ...$ The discount factor is denoted by $\delta \in (0, 1)$.
- Players, actions and profits are as before with the novelty that actions and profits are now indexed by t.
- Payoffs: $V\left(\{q_{it}\}_{t=1}^{\infty}; \{q_{-it}\}_{t=1}^{\infty}\right) = \sum_{t=0}^{\infty} \delta^t \pi(q_{it}; q_{-it})$.

2.1 Cartel as a subgame perfect equilibrium (SPE)

Consider the following strategy:

- Play $q_{it} = q^M \equiv Q^M/N$ for all i and all t if $t = 0$ and $Q_s = Q^M$ for all $s = 0, ..., t-1$ (cooperation phase).

- Play $q_{it} = q^{NE}$ otherwise (punishment phase).

2.1.1 Equilibrium payoffs

The cooperation phase payoff $V(\underline{C})$ obeys

$$(1-\delta) V(\underline{C}) = \frac{\pi^M}{N} = \left(\frac{a}{2}\right)^2 \frac{1}{bN} \tag{6}$$

by (4) [make sure you understand why].

The punishment phase payoff $V(\underline{P})$ obeys

$$(1-\delta) V(\underline{P}) = \pi^{NE} = \left(\frac{a}{N+1}\right)^2 \frac{1}{b} \tag{7}$$

by (5).

2.1.2 Deviation payoffs

The optimal deviation payoff $V(\underline{D})$ in the cooperation phase is

$$V(\underline{D}) = \pi^D + \delta V(\underline{P}), \tag{8}$$

but what is π^D? The optimal (unilateral) deviation when the other players are playing q^M is

$$q^D \equiv BRF_i\left(q^M\right) = \frac{N+1}{N}\frac{a}{4b}$$

and the corresponding one-time profit is

$$\begin{aligned} \pi^D &\equiv \pi\left(q^D; q^M\right) \\ &= \left(\frac{N+1}{N}\frac{a}{4}\right)^2 \frac{1}{b} \\ &= \frac{1}{N}\left(\frac{N+1}{2}\right)^2 \frac{\pi^M}{N}, \end{aligned}$$

where the last equality follows from (4); thus π^D is larger than π^M/N by $N \geq 1$, which is intuitive [if you don't understand why, think harder about it].

The optimal (unilateral) 'deviation' when the other players are playing q^{NE} is q^{NE} [can you see why?] so that the deviation payoff in the punishment phase, denoted by $V(\underline{DP})$, is $V(\underline{P})$ itself.

2.1.3 Conditions under which the cartel is a SPE

- Cooperating must be profitable, i.e. $V(\underline{D}) \leq V(\underline{C})$, which is the case if and only if

  $$\delta \geq \delta_{\min} \equiv 1 - \frac{4N}{N^2 + 6N + 1} \in (0, 1)$$

  by (6)-(8) [Interpret this condition]. Note that $\delta_{\min}$ is increasing in N; in the limit, when the number of firms is very large, even quite patient firms cannot sustain a cartel [provide an economic interpretation for this result]. That is to say,

  $$\lim_{N \to \infty} \delta_{\min} = 1.$$

- Punishment must be credible, i.e. $V(\underline{DP}) \leq V(\underline{P})$ must hold. Here, it holds trivially.