Monopolistic competition: the Dixit-Stiglitz-Spence model

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Abstract

The workhorse of modern Urban Economics, International Trade, Economic Growth, Macroeconomics... you name it. Each firm produces a specific variety of a differentiated good under increasing returns to scale. It faces a downward sloping residual demand curve and chooses its price monopolistically. Free-entry erodes pure profits, that is, the residual demand curve adjusts until it is tangent to the average cost curve in equilibrium.

1 The Dixit-Stiglitz-Spence model in the closed economy – the Ethier (1982) version

The set up is as follows:

- Endogenous number of input producers: n. Intermediate producers are monopolistically competitive.
- Exogenous city/region/country/world labour force: L.
- Final good freely traded, constant returns to scale, perfect competition: $p_Y = 1$.
- Production of final good requires continuum of horizontally differentiated inputs.
- CES technology: $Y = \left[\int_0^n x\left(s\right)^{\frac{\sigma-1}{\sigma}} ds\right]^{\frac{\sigma}{\sigma-1}}$, some $\sigma > 1$; this parameter is the elasticity of substitution between any two varieties.
- IRS at the firm level: $C\left(x\right) = \left(\alpha + \beta x\right) w$.
- Preferences over final good Y: $U\left(Y\right) = Y$.

1.1 Equilibrium

FINAL GOOD FIRMS minimize cost, $c(x(s) : s \in [0,n]; p(s) : s \in [0,n]) \equiv \int_0^n p(s)\, x(s) \mathrm{d}s$ subject to attaining output level Y:

$$\min_{x(s)} \int_0^n p(s)\, x(s)\, \mathrm{d}s + \lambda \left\{ Y - \left[\int_0^n x(s)^{\frac{\sigma-1}{\sigma}} \, ds \right]^{\frac{\sigma}{\sigma-1}} \right\},$$

where λ is the lagrangian multiplier. Recall that its economic interpretation is the marginal cost.

FOCs (droping the '*' in order not to burden notation):

$$0 = \frac{\partial c(\cdot)}{\partial x(s)} \equiv p(s) - \lambda \left[\frac{Y}{x(s)} \right]^{1/\sigma}$$

so that the demand for variety s is

$$x(s) = \left[\frac{p(s)}{\lambda} \right]^{-\sigma} Y.$$

Verify that σ is the elasticity of substitution between any two varieties. Rearranging and integrating yields

$$\lambda = \left[\int_0^n p(s)^{1-\sigma} \, ds \right]^{1/(1-\sigma)} \equiv P.$$

In words: the marginal cost of the final good sector is equal to the price index of intermediates. Let $m(s)$ define the market share of variety s: $m(s) \equiv p(s)^{1-\sigma} / \int_0^n p(t)^{1-\sigma} \, dt$. Multiplying both sides of the expression above for $x(s)$ by $p(s)$ and using $\lambda = P$ yields

$$\begin{aligned} p(s)\, x(s) &= \left[\frac{p(s)}{P} \right]^{1-\sigma} PY \\ &= m(s)\, p_Y Y \\ &= m(s)\, Y, \end{aligned}$$

where the second equality follows from $P = p_Y$ (which follows by perfect competition in sector Y) and the third follows from $p_Y = 1$ by our choice of numéraire. Verify *Euler's theorem* in this case (remember that the production of Y displays CRS).

MONOPOLISTICALLY COMPETITIVE INTERMEDIATE PRODUCERS set profit-maximising prices

$$p(s) = p \equiv \left(1 - \frac{1}{\sigma} \right)^{-1} \beta w,$$

all s. Observe that σ is also the perceived elasticity of demand in equilibrium. *Note the similarity with a monopoly price.*

Free-entry drives their profits to zero:

$$0 = \pi(s) \equiv \left[\frac{\beta}{\sigma - 1} x(s) - \alpha \right] w$$

so that $x(s) = x \equiv \alpha(\sigma - 1)/\beta$ for all s. Note in particular that x is independant of the city size and wage, L and w.

Full-employment of labour requires

$$L = n(\alpha + \beta x) = n\alpha\sigma$$

so that the equilibrium number of firms is proportional to city/region/country size:

$$n^* = L\frac{1}{\alpha\sigma}.$$

Perfect competition in final goods yields $\lambda = p_Y = 1$ which implies

$$\begin{aligned} 1 &= \left[\int_0^n p(s)^{1-\sigma} ds\right]^{1/(1-\sigma)} \\ &= n^{1/(1-\sigma)}\frac{\sigma\beta}{\sigma - 1}w \end{aligned}$$

so that equilibrium nominal and real wages are

$$w^* = \frac{\sigma - 1}{\sigma\beta}\left(\frac{L}{\alpha\sigma}\right)^{1/(\sigma-1)}.$$

Finally, aggregate production is equal to $Y = n^{\sigma/(\sigma-1)}x$ so that, in equilibrium,

$$Y^* = \alpha\frac{\sigma - 1}{\beta}\left(\frac{L}{\alpha\sigma}\right)^{1+\frac{1}{\sigma-1}}.$$

1.2 Agglomeration economies

Output per capita is increasing in city size, L, by inspection:

$$\left(\frac{Y}{L}\right)^* = \frac{\sigma - 1}{\sigma\beta}\left(\frac{L}{\alpha\sigma}\right)^{1/(\sigma-1)}.$$

Worker productivity (as measured by their wage w^*) is also increasing in city size: indeed, recall that

$$w^* = \frac{\sigma - 1}{\sigma\beta}\left(\frac{L}{\alpha\sigma}\right)^{1/(\sigma-1)}.$$

Ouput per capita and worker productivity are increasing in city size because larger cities enable final good producers to share a larger pool of intermediate producers – an *extensive margin*. This is a striking result: *the combination of increasing returns to scale at the firm level and free entry gives rise to aggregate scale economies.*

2 International trade and the gains from trade – the Krugman (1979)/Ethier (1982) model

There are several countries in the world economy, indexed by $c = 1, ..., C$. In autarky, larger countries command higher (real) wages (immediate from the result above).

2.1 From no trade to free trade

All countries were in autarky in the previous subsection. Here we consider a world economy in which both final and intermediate goods are freely traded. In this model, opening up to trade is like increasing the size of the world and all countries benefit from an increase in productivity. The smallest countries benefit the most. To see all this, note that

$$w_c^a = \frac{\sigma - 1}{\sigma\beta}\left(\frac{L_c}{\alpha\sigma}\right)^{1/(\sigma-1)}$$

at the autarky equilibrium and

$$w_c^{ft} = w^{ft} \equiv \frac{\sigma - 1}{\sigma\beta}\left(\frac{L^{World}}{\alpha\sigma}\right)^{1/(\sigma-1)}$$

at the free trade equilibrium, with $L^{World} \equiv \sum_{c=1}^{C} L_c$ (note that the equilibrium wage is the same in all countries; can you figure out why?). As a result, the log-difference in real wages is given by

$$\frac{w^{ft}}{w_c^a} = \left(\frac{L^{World}}{L_c}\right)^{1/(\sigma-1)},$$

which establishes that the gains from trade are largest for the smallest countries. Intuitively, the larger a country, the closer it is to being the world on its own and the less it has to gain from market integration.

2.2 From some trade to freer trade

The final good is freely traded but shipping intermediates across international borders incurs an iceberg cost of $\tau > 1$. That is, τ units need to be shipped for one unit to arrive at destination ($\tau \to +\infty$ corresponds to autarky, $\tau = 1$ corresponds to free trade in intermediate inputs). In this case,

$$P_1^{1-\sigma} = \frac{\sigma\beta}{\sigma - 1}\left[n_1 w_1^{1-\sigma} + \tau^{1-\sigma}\sum_{c=2}^{C} n_c w_c^{1-\sigma}\right],$$

and symmetrically for $P_2, ..., P_c$. Using the full-employment conditions $L_c = n_c\alpha\sigma$ and the no-arbitrage condition $P_1 = P_2 = ... = 1$ yield (this expression holds for $\tau \neq 1$)

$$w_c(\tau) = \frac{\sigma - 1}{\sigma\beta}\left\{\left[1 + (C-1)\tau^{1-\sigma}\right]\frac{L_c}{\alpha\sigma}\right\}^{1/(\sigma-1)}$$

so that a fall in τ results in higher wages everywhere. Note further that

$$\begin{aligned}\frac{\mathrm{d}\ln w_c}{\mathrm{d}\ln\tau} &= -\frac{(C-1)\,\tau^{1-\sigma}}{1+(C-1)\,\tau^{1-\sigma}} \\ &= -\left[1+\frac{1}{(C-1)\,\tau^{1-\sigma}}\right]^{-1},\end{aligned}$$

that is, all countries benefit from freer trade (i.e. a fall in τ) in the same proportion. Finally, note that

$$\frac{\mathrm{d}^2 w_c}{\mathrm{d}\tau\mathrm{d}L_c} = \left(\frac{1}{\alpha\sigma}\right)^{1/(\sigma-1)}\frac{C-1}{\sigma\beta}\tau^{-\sigma}\left\{\left[1+(C-1)\,\tau^{\sigma-1}\right]L_c\right\}^{-1+1/(\sigma-1)} > 0,$$

namely, the rise in wages following a fall in τ is higher in small countries than in large countries.