Trade-in-goods and trade-in-tasks: An integrating framework

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ABSTRACT

We introduce a simple but flexible analytical framework in which both trade in goods and trade in tasks arise. We use this framework to provide versions of the gains-from-trade and the famous four HO theorems (Heckscher-Ohlin, factor-price-equalisation, Stolper-Samuelson, and Rybczynski) that apply to this environment. We extend our framework to accommodate monopolistic competition and two-way offshoring and to integrate theoretical results of the early offshoring literature.

1. Introduction

A growing list of economists argues that the nature of international trade is changing in important ways. Instead of simply creating more trade in goods, global integration is increasingly marked by trade of intermediate goods and services, a.k.a. ‘fragmentation’, ‘offshoring’ or ‘task trade’. The importance of this trade has been clarified with new data sets on ‘value added’ trade, which remove the double counting from ‘gross’ trade as traditionally measured by customs statistics that arises as intermediates cross borders on their own and are embodied in further processed goods (Johnson and Noguera 2012, Koopman, Powers, Wang, and Wei 2013).

In this paper, we introduce a simple but flexible analytical framework in which both trade in goods and trade in tasks arise endogenously in response to exogenous changes in the cost of moving goods and services. We then use this framework to provide

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versions of the traditional four theorems (Heckscher-Ohlin-Vanek, factor-price-equalisation, Stolper-Samuelson, and Rybczynski) that are valid when both trade in goods and trade in tasks occur. For example, in terms of the ‘gross’ versus ‘value added’ trade distinction, our versions of the Heckscher-Ohlin-Vanek theorem link gross trade in final goods to national endowments in a way encompasses the possibility of task trade. Finally, we show how specific assumptions in the offshoring literature simplify the analysis and lead to specific and strong results in our framework as well.

Integrating trade-in-tasks into the trade-in-goods literature is important for at least three reasons. First, a substantial fraction of world output remains traded across international boundaries even after the trade collapse of 2008-2009, and trade in components and intermediate services represents a growing fraction of this trade (WTO 2008). It is thus important to study these facts jointly. Second, this trend has elicited a substantial number of theoretical contributions, reviewed in some detail below, that is marked by a wide range of cases where outcomes seemingly contradict standard trade theory results. We show that this arises because task trade and trade in goods display interesting similarities but task trade differs in that it typically involves some kind of technology transfer that is akin to *product augmenting technical change* (Dixit and Norman, 1980, chapter 5). Finally, this technological transfer interacts with traditional sources of comparative advantage and these interactions require that the traditional four theorems be amended; specifically, trade in tasks becomes a source of comparative advantage in final goods (so that the Heckscher-Ohlin-Vanek and the factor-price-equalisation theorems break down in their standard formulation), has important wage effects beyond and above the Stolper-Samuelson effects, and implies some kind of shadow migration that exhibits Rybczynski effects on production and trade patterns for final goods.

Formally, we extend the traditional Heckscher-Ohlin-Vanek (henceforth HO) framework to allow for trade in tasks in the wake of Grossman and Rossi-Hansberg (2008), henceforth GRH. In the model, there are two countries, Home and Foreign, producing final goods by combining tasks, each of which involves primary factors of production. Importantly, Home has a Hicks-neutral technological superiority so its wages are higher even though there is conditional factor price equalisation absent offshoring (under the usual regularity conditions). This wage gap drives the offshoring when trade in some, but not all, tasks becomes feasible.
This has four noteworthy consequences. First, the reduction in Home firm production costs due to offshoring is analogue to a technical change (the ‘productivity effect’ in GRH) and in the competitive equilibrium at least some – and possibly all – Home factor prices must rise; this analogy and this finding are central to the analysis of GRH. Second, offshoring is akin to ‘shadow migration’ – i.e. it is as if foreign factors migrated to the offshoring nation and we show that the quantity effects on production and trade in final goods follow the logic of the Rybczynski theorem; to the best of our knowledge, this analogy is novel. Third, the combination of heterogeneous factor intensities at the task level implies heterogeneous task intensities at the final good level so that Home’s effective technological superiority is no longer Hicks-neutral. Put differently, trade patterns in final goods are now governed by a combination of Ricardian and HO forces.

The four traditional HO theorems break down in this environment. A key contribution of our paper is to transform the HO equations using shadow endowments instead of actual ones and to establish analogues to the traditional HO theorems in this modified version. Finally, starting from an equilibrium with trade in goods but no task trade, allowing for offshoring has ambiguous welfare effects because it affects the terms-of-trade. In this respect, increasing access to trade at the extensive margin is similar to increasing it at the intensive margin: more trade unambiguously raises the welfare of a country only starting from autarky.\(^2\)

Our framework departs from the offshoring literature of the 1990s and early 2000s (which we review below) by making two specific assumptions – arbitrage opportunities arising from Hicks-neutral technology differences (an assumption we maintain throughout) and Leontief technologies (an assumption we sometimes relax). This enables us to obtain strong analytical results and precise conditions to sign the factor price, output, and trade effects of offshoring, another key novelty of our paper. Several of these effects are comparable to results uncovered by this early literature and we also show under which conditions they also arise in our setting. In this sense, our framework enables us to integrate this literature within the standard HO toolkit.

\(^2\) The gains-from-trade results in our paper were simultaneously and independently developed by Markusen (2013). Both papers use the methodology introduced by Dixit (1985). Markusen then applies the analysis to the positive models developed by Markusen and Venables (2007) and GRH.
Additionally, we show that trade-in-tasks creates intraindustry trade in a Walrasian economy.

We then depart further from the early offshoring literature by assuming that task trade implies technology diffusion. In this environment, offshoring has novel price and quantity effects on the host country, Foreign. Finally, we extend the basic framework to accommodate monopolistic competition and two-way offshoring/trade-in-tasks.

1.1. The theoretical literature

Integrating trade in tasks and trade in goods in a unified and analytically tractable setting is the main innovation of our paper. It builds on and expands a large body of theoretical literature in trade theory. The early HO theory incorporates trade in intermediate goods (Batra and Casas 1973, Woodland 1977, Dixit and Grossman 1982, and Helpman 1984) and the 1990s saw a number of informal analyses of fragmentation as well as some formal modelling (Deardorff 1998a, b, and Venables 1999). Trade-in-tasks issues, however, were more recently crystallised by Kohler (2004a), Markusen (2006), Antrás et al. (2006), and Grossman and Rossi-Hansberg (2006, 2008).

The most commonly cited reference in the early offshoring/fragmentation literature is the diagrammatic analysis of Jones and Kierzkowski (1990), which seems to be the first to leverage the insight that fragmentation acts as technological progress and should therefore be expected to have complex wage effects. The ensuing line of modelling typically works with small open economies where fragmentation occurs in one sector and in one direction. The focus of the analysis is firmly on wage effects. Jones and Kierzkowski (1990), for instance, argue that workers whose jobs are ‘lost’ to offshoring may, somewhat paradoxically, see their wages rise in some special cases.

Among the mathematical formalisations of fragmentation, Deardorff (1998a,b) studies fragmentation in a multi-cone HO model where cost-saving offshoring is driven by non-factor price equalisation. The focus is on factor prices and showing that trade-in-tasks need not foster wage convergence. Venables (1999) works with a 2x2x2 HO model where offshoring is cost saving due to non-factor price equalisation arising from a factor-intensity reversal. Fragmentation occurs in one industry and in one direction.

He uses numerical simulations and Lerner-Pearce diagrammatic analysis to study examples where trade-in-tasks produces wage convergence and divergence. Kohler (2004a) works with a small-open-economy specific-factor model where fragmentation occurs in one sector. The focus is on the reward to the specific capital that moves offshore when fragmentation occurs and on the overall welfare effects on the home nation. Markusen (2006) works with a multi-cone HO model that he simulates numerically assuming that fragmentation occurs in the skill-intensive sector and the fragment is of middle skill-intensity. He typically finds that skilled workers gain. Kohler (2004b) works with a small open economy where fragmentation/offshoring can only happen in one sector, using the Dixit and Grossman (1982) model with a continuum of intermediate goods; he shows that cheaper offshoring raises or lowers factor prices according to the relative factor intensity of the two sectors and the fragments offshored. We add production and trade effects to this literature. By imposing additional structure on the model, we are also able to repeat some of its quantitative findings by analytic means.

More recently, Grossman and Rossi-Hansberg (2006, 2008) present a perfect competition model where two final-goods are produced using two continuums of tasks, each employing only one type of labour. Offshoring arises endogenously and the range of tasks offshored varies continuously with the cost of offshoring. The paper formalises the analogy between offshoring and technological change (the ‘productivity effect’) showing that trade-in-tasks, unlike trade-in-goods, can generate gains for all factors in the offshoring nation. The paper establishes necessary and sufficient conditions for wage-changes in the two-factor-two-good small open economy case. It also explores the novel ‘labour supply effect’ that influences wages when there are more factors than goods.

There also exists a string of recent offshoring papers that are not encapsulated by our integrating approach. Rodriguez-Clare (2010) embodies the GRH approach in a Ricardian model à la Eaton and Kortum (2002). He studies the impact of trade-in-tasks on the gains from trade for the home and host nations. Global welfare rises due to offshoring’s productivity effect, but terms-of-trade effect can mean that the home nation loses despite this. Antràs, Garicano and Rossi-Hansberg (2006, 2008) propose a model in which all tasks are potentially offshorable. The focus is on the formation, composition and size of (cross-border) teams when workers have different abilities.
(skills), and countries have different skill endowments. Among other results, they show that improved communication technology yields larger teams and larger wage inequalities. Their model also provides a trade-induced explanation for the rise in returns to skills. Fujita and Thisse (2006) and Robert-Nicoud (2008) study how offshoring interacts with agglomeration forces in ‘new economic geography’ settings while Antràs and Staiger (2012) analyse the optimal design of trade agreements in the presence of offshore outsourcing.

1.2. Organisation of paper

The next section introduces notation and a slightly modified HO model of trade in final goods. Section 3 adds offshoring/trade-in-tasks and Section 4 solves for the equilibrium. Section 5 uses our framework to integrate previous results of the fragmentation/offshoring/task-trade literature. Section 6 extends the framework in various directions and Section 7 concludes.

2. Trade in goods

To fix notation, this section presents an HO model modified slightly à la Trefler (1993); the modification creates an incentive for offshoring when the possibility arises in Section 3.

There are two countries, Home and Foreign (Foreign variables distinguished by asterisks), $F$ factors of production, and $I$ perfectly competitive industries ($f = 1, ..., F$ and $i = 1, ..., I$ index factors and industries respectively). The factor price, goods price, factor endowment, production, consumption and import vectors are denoted by $\{w_f\}_F$, $\{p_i\}_I$, $\{V_i\}_F$, $\{X_i\}_I$, $\{C_i\}_I$ and $\{M_i\}_I$. The $I \times F$ matrix $A(w) \equiv \{a_{fi}(w)\}$ and its transpose $A^T$ summarise Home’s constant returns technology with typical element $a_{fi}$ giving the cost-minimizing input requirement of factor $f$ in industry $i$ as a function of $w$: $a_{fi} = \frac{\partial c_i(w)}{\partial w_f}$, where $c_i$ denotes the unit cost function in industry $i$; let $c(w) \equiv \{c_i(w)\}$. Tastes are homothetic and identical across nations.

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4 Vectors and matrices are denoted by bold letters; variables and parameters by italics, and $Z > N$ means that each element of $Z$ exceeds the corresponding element of $N$. 

We adopt standard regularity conditions to ensure that a unique equilibrium exists with diversified production throughout the paper:

**Assumption 1 (diversified production).** Home and Foreign produce positive outputs in all sectors at any equilibrium we study in the paper.\(^5\)

Our departure from the standard model is that Home is technically superior in the Hicks-neutral sense:

**Assumption 2.1 (homothetic technologies).** Foreign unit cost functions are \(\gamma > 1\) times higher than Home’s, i.e. \(\mathbf{c}^* (\cdot) = \gamma \mathbf{c}(\cdot)\).

Such Hicks-neutral technology differences do not create Ricardian motives for trade. As is well known, the model can be mechanically transformed into a standard HO model by defining Foreign factor supplies in ‘effective units’, i.e. dividing \(V_j^*\) by the technology gap \(\gamma\). We denote effective units of factors by “~”, so the world factor endowment in effective units is \(\bar{V}^w \equiv V + V^*/\gamma\).

The autarky equilibria are characterised by market-clearing conditions \(M^* = 0\) and \(M = 0\) as well as \(I\) pricing conditions and \(F\) employment conditions in each nation, which in familiar notation are:

\[
\mathbf{p} = \mathbf{c} = \mathbf{A} \mathbf{w}, \quad \mathbf{p}^* = \gamma \mathbf{c}^* = \gamma \mathbf{A}^* \mathbf{w}^*; \quad \mathbf{V} = \mathbf{A}^\top \mathbf{X}, \quad \mathbf{V}^* = \gamma \mathbf{A}^{*\top} \mathbf{X}^* \tag{1}
\]

where the arguments are suppressed for simplicity, so \(\mathbf{c}(\mathbf{w}), \mathbf{c}(\mathbf{w}^*), \mathbf{A}(\mathbf{w})\) and \(\mathbf{A}(\mathbf{w}^*)\) are written as \(\mathbf{c}, \mathbf{c}^*, \mathbf{A}\) and \(\mathbf{A}^*\), respectively.

At the free-trade equilibrium, goods prices are equalised (law of one price), goods-markets clear globally (\(M^* + M = 0\)), and equation (1) characterises the equilibrium but with a common \(\mathbf{p}\). Under standard regularity conditions, equilibrium production and price vectors are strictly positive.\(^6\)

Throughout the paper, we assume \(\mathbf{A}\) is invertible in which case the law-of-one-price can, and assuming no factor intensity reversals, must imply effective factor price

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\(^5\) The condition is that the \(\mathbf{V}\)’s lie in the Deardoff (1994) lens, i.e. the space spanned by the columns of \(\mathbf{A}^\top\) evaluated at equilibrium factor prices.

\(^6\) See the appendix of our working paper for necessary and sufficient conditions for existence (http://www.dagliano.unimi.it/media/wp2008_250.pdf).
equalisation (FPE), i.e. $w = \gamma w^*$ – a fact established by simple manipulations of (1) using the fact that $c = c^*$ if and only if $w = \gamma w^*$. With homothetic preferences, a common world price vector $p$, and $A = A^*$, the effective-factor-content of $C$ must be equal to $s\tilde{V}^*$, where $s$ is Home’s share of world income. The factor content of Home production is $V$, so the pattern of trade must respect the HO theorems:

$$A^T M = s\tilde{V}^* - V$$

The third and fourth standard theorems consider the impact on $w$ of an exogenous variation in $p$ (Stolper-Samuelson theorem) and the impact on $X$ of an exogenous variation of $V$ (Rybczynski theorem); these follow from simple manipulations of (1) given that $A = A^*$ under free trade.

The standard gains-from-trade (GFT) theorem states that (i) some trade is better than none – ignoring intra-national distribution issues (Ohyama 1972, Smith 1982, Dixit 1985) and (ii) more trade is desirable if terms of trade do not worsen. As the analogue for part (ii) of the GFT theorem holds for trade-in-tasks (see Proposition 2 below), we review why it holds for trade-in-goods. By revealed preference arguments (Samuelson 1939, 1962, Kemp 1962), an equilibrium is preferred to another if the inferior equilibrium’s consumption vector is affordable at the preferred equilibrium’s prices. Denoting Home’s autarky consumption vector as $C_a$, the trade equilibrium is preferred by Home if $p(C - C_a) \geq 0$. Using the definition of $M$, the condition can be written as $p(M - M_a) + p(X - X_a) \geq 0$. This inequality holds because: (i) the first term is zero due to balanced trade ($pM = 0$) and by definition of autarky ($M_a = 0$), and (ii) profit maximisation by Home firms implies the second term is positive. A symmetric result holds for Foreign, so both nations gain from trade. Furthermore, gains from partial liberalisation arise if terms of trade improve or do not change (Dixit 1985).

3. Trade in tasks

This section modifies the model to allow trade in tasks.

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7 To see why effective FPE easily holds when $I \geq F$, we write $\gamma c(w^*) = c(\gamma w^*) = c(w)$. The former equality follows from the fact that $c(\cdot)$ is homogenous of degree 1 in factor prices and the latter follows from the law of one price, $p^* = p$. The implications of relaxing the assumption $I \geq F$ are not innocuous but well understood (Ethier 1984).
3.1. Framework

Production in industry $I$ involves $N_i$ tasks indexed by $i = 1, \ldots, N_i$, $N_i \geq 2$. Tasks are either segments of the physical production process (so the task’s output is an intermediate good, say wheels) or a slice of the necessary factor inputs (so the task’s output is a productive service, say accounting services). In the model described above, all tasks were bundled into the unit cost functions $c_i(w)$, $i = 1, \ldots, I$. This implicitly assumed that all tasks in a given production process had to be performed in a single nation. Here we consider an exogenous change that allows the production process to function even when tasks are spatially unbundled – thus opening the door to offshore production and the attendant trade-in-tasks. Let $N_i$ be the number of tasks need to be performed to produce good $i$ and $c_{it}$ denote the cost of producing the amount of task $t$ necessary to produce one unit of good $i$; in keeping with the literature (e.g. GRH), we assume that all tasks are assembled using a Leontief technology so that $c_i(\cdot) = \sum_{t=1}^{N_i} c_{it}(\cdot)$. More specifically, each task involves a non-negative quantity of each factor $f$, so with constant returns, $a_{ft}$ can be written as the sum of task-level coefficients:

$$a_{ft}(w) = \sum_{t=1}^{N_i} a_{ft}(w); \quad \forall f = 1, \ldots, F; \quad i = 1, \ldots, I \quad (3)$$

where $a_{ft}$ denotes the unit input requirement of factor $f$ for task $t$ in sector $i$. This allows substitutability of factors in the performance of individual tasks, but not of tasks. For, example if making wheels is one task then each car requires exactly four wheels; extra wheels cannot be substituted for the engine. A key additional assumption is:

**Assumption 2.2 (firm-specific technologies).** Firms that offshore a task can do so using the technology of their own nation.\(^8\)

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\(^8\) The concept of what constitutes a firm does not seat well with our otherwise Walrasian model and we use it as a shortcut for the common idea in the FDI literature that technologies may be implemented in countries different from those in which they have originated, as in e.g. Ramondo and Rodriguez-Clare (2010). Section 6 shows that our results all got through in a monopolistic competition trade model where firms are well-defined; here we stick with the HO setting to improve comparison with the four theorems.
This makes offshoring economical despite effective FPE. Home firms can combine their superior technology with lower Foreign factor prices, so offshoring tasks to Foreign may be economic; Foreign-to-Home offshoring will never be economic. One interpretation of this assumption is that Foreign workers are themselves as productive or as well educated as Home workers but that Foreign technology or management practices are inferior to Home’s (Bloom and Van Reenen 2007).  

In order to be concrete about the exogenous changes that allow trade-in-tasks, we introduce “coordination costs” i.e. the cost of exchanging information necessary to coordinate various tasks into a single production process. Routine tasks, which are easily codified, are cheaper to offshore than complex tasks that require frequent face-to-face interactions.

3.2. Free trade in tasks

To integrate trade-in-tasks results with trade-in-goods theory, we focus on extreme changes of these coordination costs. Without further loss of generality, we set \( N_f = 2 \).

For the routine tasks, which we call type-1 tasks, the switch is from prohibitive to zero. For complex tasks, type-2 tasks, the coordination costs remain prohibitive. That is to say, task \( t = 1 \) is the set of all tasks that can be offshored at zero coordination cost and task \( t = 2 \) is the set of tasks that are prohibitively expensive to offshore. By the usual cost-savings logic, all Home production of type-1 tasks is offshored to Foreign (assuming standard regularity conditions that ensure diversified production in both economies).

Formally:

**Lemma 1 (trade-in-tasks occurs).** Under regularity conditions that assure diversified production, all type-1 tasks are offshored from Home to Foreign in the trade-in-tasks equilibrium.

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9 Assuming that offshoring involves a convex combination of domestic and foreign technologies would make the model more realistic but would not change our qualitative results provided that this combination is the same for all tasks. More interesting and far-reaching is the idea that offshoring also implies coordinating production over several, distant locations and possibly among different firms; both involve important costs. If such costs vary by task then the implications on factor prices and on quantities produced and traded are much more complex.

10 The appendix of our working paper provides exact necessary and sufficient conditions for diversification in the 2x2x2 version of the model ([http://www.dagliano.unimi.it/media/wp2008_250.pdf](http://www.dagliano.unimi.it/media/wp2008_250.pdf)).
Proof. Suppose first that trade in type-1 tasks was possible but none occurred in equilibrium. As this prospective equilibrium is identical to the trade-in-goods equilibrium, \( w \) would equal \( \gamma w^* \), so by Assumption 2 an atomistic firm deviating from the prospective equilibrium would reduce costs by offshoring its type-1 tasks; the per-unit-output cost reduction would be equal to
\[
1_i \left( w - w^* \right) = c_i \left( \gamma w^* - c_i \left( w^* \right) > 0 \right.
\]
The resulting pure profit contradicts the definition of a competitive equilibrium, so some trade-in-tasks occurs; factor prices adjust so that
\[
c_i \left( w_o \right) - c_i \left( w^*_o \right) \geq 0 ; \quad \forall \ i = 1,\ldots,I
\]
in the trade-in-task equilibrium (the subscript ‘O’ stands for ‘Offshoring equilibrium’).

Suppose next that some Home firms do not offshore all type-1 tasks. Such firms are foregoing the cost-saving opportunity and thus earn negative profits when competing with firms that do, except in the knife-edge case with
\[
c_i \left( w_o \right) = c_i \left( w^*_o \right),
\]
in which case they earn zero profit and are indifferent. To see this, note that
\[
0 = p_o^* - p_o = \gamma c \left( w^*_o \right) - \left[ c_i \left( w^*_o \right) + c_i \left( w_o \right) \right] \leq \gamma c \left( w^*_o \right) - \left[ c_i \left( w_o \right) + c_i \left( w_o \right) \right] = \gamma c \left( w^*_o \right) - c \left( w_o \right),
\]
where the first equality follows by the law of one price, the second one follows from diversification as well as from the zero profit conditions of Foreign firms and those of Home firms that offshore task 1, and the inequality follows from (4): it applies to Home firms that don’t offshore all of task 1. Thus \( c \left( w_o \right) \geq p_o \), these firms cannot compete with either Foreign firms or Home firms that do offshore. QED.

Given Lemma 1, Home’s pricing and employment equations reflect Foreign-factor usage (for type-1 tasks), while Foreign’s pricing condition is unaffected (Foreign firms continue to use Foreign technology and pay Foreign wages). Foreign’s employment condition, however, reflects offshoring employment. In the no-local-sales case, all offshore task production is re-imported to Home, so Foreign employment in the offshore sector is proportional to Home’s production vector. Formally, using the subscript ‘O’ (for ‘offshoring’) to indicate trade-in-task equilibrium variables, we may
integrate trade-in-tasks with the trade-in-goods framework and write the analogue to equation (1) as:

\[ p_o = c_1(w_o) + c_i(w_o^*) = (A_o - A_i)w_o + A_i^*w_o^*. \]
\[ p_o = \gamma c(w_o^*) = \gamma A_i^*w_o^*. \]
\[ V = (A_o^T - A_i^T)X_o, \quad V^* = \gamma A_o^T X_o^* + A_i^T X_o. \]  

(5)

where \( A_o \equiv A(w_o), A_i \equiv \{a_{ji}(w_o)\}, A_i^* \equiv \{a_{ji}(w_o^*)\}, \) and \( A_o^* \equiv A(w_o^*). \)

Above, the first line and second line expressions are the marginal cost pricing in Home and Foreign, respectively, while the third line collects the full employment conditions for both countries. In effect, Home and Foreign primary factors of production no longer use the same homothetic technologies (Home factors do not convey type-1 tasks); in this sense, task trade creates Ricardian motives for trade in final goods in equilibrium (more on this in Section 4 below) and blurs the HO logic and its analytic elegance.

### 3.3. Leontief technologies

As we shall see below, working with Leontief technologies enables us to obtain sharper results in many instances, so we sometimes impose the following assumption:

**Assumption 3 (Leontief technologies).** Input-output coefficients of all factors, tasks, and goods are invariant in factor prices.\(^{12}\)

As a result, \( A_o^* = A_o = A \) and \( A_i^* = A_i \). Using these, we may rewrite the pricing equations and the full-employment conditions of equation (5) as

\[ p_o = (A - A_i)w_o + A_i^*w_o^*, \quad p_o = \gamma Aw_o^*. \]
\[ V = (A^T - A_i^T)X_o, \quad V^* = \gamma A^T X_o^* + A_i^T X_o. \]  

(6)

These expressions are an approximation of the general case (i.e. with flexible technology) in two cases: (i) when the scale of offshoring is modest, so factor-prices changes are modest and \( A \) changes are second-order-small by the envelope theorem; and (ii) when the technology is such that the \( a_{ji} \)'s are not very sensitive to factor prices.

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\(^{11}\) In writing (5) we maintain the assumption of diversified production, though offshoring brings additional motives for specialisation. See the appendix of our 2008 working paper for details (http://www.dagliano.unimi.it/media/wp2008_250.pdf).

\(^{12}\) We conduct part of the analysis replacing this assumption by Taylor expansions around the no-task-trade free-trade-in-goods equilibrium in an earlier version of this paper (http://www.unige.ch/ses/dsec/research/wps/11112_baldinrobertnicoud.pdf).
4. Equilibrium characterization and the shadow migration approach

We modify the GFT theorem to account for trade-in-tasks and we show that the HO and FPE theorems break down with trade in tasks in subsection 4.1; we then introduce our shadow migration approach to restore the tractability of the HO framework in subsections 4.2 (in general) and 4.3 (with Leontief technologies). Henceforth, we use ‘trade-in-goods’ to mean trade in final goods without trade in tasks (unless we specify otherwise), namely, trade in the output of industries \( i = 1, \ldots, F \) only.

4.1. Trade-in-tasks and the GFT, FPE and HO theorems

We start with the GFT theorem. Allowing for offshoring has ambiguous welfare effects because it affects the terms-of-trade – a result established by application of the Dixit (1985) technique for comparing restricted trading equilibria. In this respect, increasing access to trade at the extensive margin is similar to increasing it at the intensive margin (see also Markusen 2013): more trade unambiguously raises welfare only starting from autarky. Under our Walrasian assumptions, the cost of combining the output of type-1 and type-2 sets of tasks into a consumable good is zero, so we can readily apply Dixit’s result. We think of there being 2\( I \) goods (the two sets of tasks for each of the \( I \) goods) whose ‘shadow prices’ are the actual marginal production costs (i.e. including offshoring in the trade-in-tasks equilibrium).\(^{13}\) The relevant GFT condition is therefore

\[
0 \geq \bar{O} - \bar{C} - \bar{C}
\]

where bars indicate the artificially extended vectors, and the price vector consists of the shadow prices (marginal costs). As before, this implies

\[
(\bar{p} - \bar{p}_o)\bar{M} + \bar{p}_o(\bar{X}_o - \bar{X}) \geq 0
\]

due the definition of imports and the fact that trade balance implies

\[
\bar{p}\bar{M} = \bar{p}_o\bar{M}_o = 0.
\]

Profit maximisation assures that \( \bar{p}_o(\bar{X}_o - \bar{X}) \) is positive, but the term \( (\bar{p} - \bar{p}_o)\bar{M} \) can be positive or negative; indeed, this is the Laspeyres index of Home’s terms-of-trade loss when trade-in-tasks is allowed. Offshoring could, for example, boost global production of Home exports more than Home’s imports, engendering a terms-of-trade loss. Relative output, however, could

\(^{13}\) In his ‘fragmentation’ paper, Kohler (2003) uses the term ‘effective price’ instead of ‘shadow price’ but both terms recover the same concept.
fall in the opposite direction, so the terms-of-trade impact is ambiguous. Isomorphic reasoning implies Foreign GFT are also not assured, but the zero-sum nature of terms-of-trade effects alerts us to the fact that at least one nation must gain from offshoring.

If goods prices are unaffected by trade-in-tasks, Home gains and Foreign is unaffected (Foreign is also strictly better off in the model of Section 6.1). Formally (proof in the text), we write:

**Proposition 2 (ambiguous GFT from trade-in-tasks).** (i) Trade-in-tasks is Pareto improving if terms-of-trade are unaffected (i.e. if $\bar{p}_o = \bar{p}$); (ii) global welfare rises in all cases as terms-of-trade effects disappear at the global level; (iii) a necessary condition for a nation to lose is that it experiences a terms-of-trade loss.

The FPE and HO theorems require homothetic preferences and technologies around the world. With offshoring, the latter actually breaks down (except in knife-edge cases). As a result, the HO and FPE theorems break down – at least in their literal sense. Starting with the FPE theorem:

**Proposition 3 (effective factor price divergence).** Unless there exists a real number $\phi$ in the unit interval such that $c_1(\cdot) = \phi c(\cdot)$ (and thus $A_1 = \phi A$), trade-in-tasks forces a divergence of (effective) factor prices. ($c_1(\cdot) = \phi c(\cdot)$ is the knife-edge case where the sets of type-1 and type-2 tasks have identical factor intensity.)

**Proof.** The proof is by contradiction. Assume that effective FPE holds, that is, $\alpha w_o = w_o^*$ for some $\alpha \in (0, 1 / \gamma)$. Then the law of one price and (5) yield

$$0 = \gamma [c_1(\alpha w_o) + c_2(w_o)] - [c_1(\alpha w_o) + c_2(w_o)]$$

$$= \alpha (\gamma - 1) c_1(w_o) - (1 - \alpha \gamma) c_2(w_o)$$

$$= \alpha (\gamma - 1) c(w_o) - (1 + \alpha) c_2(w_o),$$

where the first equality follows by the law of one price and by effective FPE, the second one by the homogeneity of decree 1 in factor prices of the unit cost functions, and the third one by the definition $c(\cdot) = c_1(\cdot) + c_2(\cdot)$. The final equality holds in either of the following cases: (i) no offshoring occurs, which violates Lemma 1; (ii) all tasks are offshorable (i.e. $c_1 = c$), which is ruled out by assumption; (iii) all factor prices are zero, which violates the zero profit condition for positive goods prices; or
(iv) the factor intensity of type-1 tasks are exactly proportional to aggregate factor intensity in each industries, i.e. $c_1(\cdot) = \phi c(\cdot)$ for $\phi = (\gamma - 1)\alpha / (1 - \alpha)$. Thus the supposition that effective FPE occurs under trade in task must be false in general. QED.

Intuition for this result is simple. As authors from Jones and Kierzkowski (1990) to GRH have argued, fragmentation-trade-in-tasks is akin to technological progress for the offshoring nation. As the new trade involves a subset of tasks and offshoring is unidirectional, the technological change is non-homothetic and this destroys effective FPE. Intuition is further served by deviating from the long-standing tradition in the fragmentation/offshoring literature by considering the case where all tasks are offshorable. In this extreme case, no goods are produced using Foreign technology as such goods would be uncompetitive with goods produced using Home technology. In short, Home technology supplants Foreign technology globally, resulting in perfect factor price equalisation.

Perhaps the most robust theoretical finding in trade theory is the HO theorem. Does this hold when trade-in-tasks as well as trade-in-goods occurs? Given homothetic preferences, Home’s consumption vector of final goods is proportional to world output, i.e. $C_o = sX_o^*$, however solving for $X_o$ and $X_o^*$ from (5):

$$MK = sX_o^w - X_o$$

$$= s\left\{\left[1 - (\gamma A_o^T)^{-1}A_1^T\right](A_o^T - A_1^T)^{-1}V + (\gamma A_o^T)^{-1}V^*\right\} - (A_o^T - A_1^T)^{-1}V \quad (7)$$

The only circumstance in which this reduces to the standard HO expression in (2) is when the offshoring matrices $A_1$ and $A_1^*$ are both zero – i.e. when no offshoring occurs. In short, given Lemma 1, we can say that the HO theorem breaks down with trade-in-tasks.

4.2. The shadow migration approach

As is clear from Proposition 3 and expressions (5) and (7), trade-in-tasks ruins much of the elegance and tractability of the HO model, and this arises for four reasons. First, by Lemma 1, Home and Foreign choose different positions on their isoquants so the $A$

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14 Note that we have written (7) for imports in final goods only. Adding task imports would only reinforce the point.
matrices are not proportional. Second, even if techniques were invariant to factor prices (Leontief), Home and Foreign goods are effectively produced with different technologies where the differences are non-homothetic in general. Third, some Foreign factors use Foreign technology while others use Home technology. Finally, Home exports in final goods embody some Foreign primary factors. Each problem disrupts the elegant flow of HO logic.

A key contribution of our paper is to suggest a transformation of the model that restores much of the HO elegance and does so in a way that enables us to integrate trade-in-tasks theory into the received body of trade-in-goods theory. It also allows us to integrate the wide range of special cases considered in the offshoring literature. The transformation turns on the insight that offshoring is like ‘shadow migration’. That is, Home firms employ Foreign factors to produce tasks using Home technology, so offshoring affects the equilibrium in a way akin to migration of Foreign factors to Home assuming the migrated factors were paid foreign wages rather than Home wages.

The shadow-migration transformation has two manifestations – one for quantities and one for prices – with each involving the introduction of a new vector. Both are potentially observable given modern datasets as they require only information on the offshored production (in addition to the usual information of w’s, X’s and A’s). We start by transforming the pricing equations of (5) as

\[ \mathbf{p}_o + \tilde{S} = \mathbf{c}(\mathbf{w}_o), \quad \mathbf{p}_o = \mathbf{c}(\gamma \mathbf{w}_o^*); \quad \tilde{S}_i = c_i(\mathbf{w}_o) - c_i(\mathbf{w}_o^*) \geq 0, \quad (8) \]

where the zero-profit condition pertaining to Foreign makes use of the homogeneity of degree one of the unit cost function and \( \tilde{S} \) equals the difference between the cost of performing the offshored tasks in Home and Foreign: Home firms save money because they pay the shadow migrants at their lower Foreign prices. In this way, it follows by inspection that the departure from effective FPE tends to be increasing in the cost-savings. It also follows by inspection of (8) that we may use Jones’ (1965) algebra that offshoring is analogous to ‘product augmenting technical change’ (Dixit and Norman 1980, Chapter 5). This is a central finding of GRH, who call this the ‘productivity effect’ of trade-in-tasks.
**Proposition 4 (trade-in-tasks analogue to FPE theorem).** Starting from the trade-in-goods equilibrium, allowing trade-in-tasks produces a divergence in effective factor prices that is proportional to the value of the cost-saving stemming from trade-in-tasks.

**Proof.** The proof is by inspection of (8). Under the trade-in-goods equilibrium, effective factor price equalisation, \( w = \gamma w^* \), holds. Trade in tasks changes all goods and factor prices, in general, but the effective factor price gap is equal to
\[
\begin{align*}
\{\n\gamma w_o^* - \gamma w_o^* = c(p_o + \tilde{S})^{-1} - c(p_o)^{-1},
\end{align*}
\]
which is in different from zero in general. \(QED\).

An implication, whose proof is by inspection of (8), is:

**Corollary 4.1 (shadow migration not necessarily a substitute for real migration).**

From Proposition 5, shadow migration can widen or narrow the international wage gap for each type of labour, so offshoring may increase or decrease the pressure for real migration.

We obtain sharp results by imposing Assumption 3 (Leontief technologies) and we turn to this next.

**4.3. Leontief technologies**

The offshoring cost-saving vector \( \tilde{S} \), denoted as \( S \) in the Leontief case, is equal to
\[
S = A_1(w_o - w_o^*).
\]
We may thus rewrite the pricing equations in (6) and (8) as
\[
\begin{align*}
p_o + S &= Aw_o, \\
p_o &= \gamma Aw_o^*
\end{align*}
\]
for Home and Foreign, respectively. Observe the latter expression is isomorphic to the pricing equation for Foreign in equation (1), which pertains to the equilibrium with free trade-in-goods but no trade-in-tasks, while the former emphasizes one more time that offshoring is analogous to product augmenting technical change for Home firms.

Turning to quantities, we define the shadow migration vector, denoted as \( \Delta V \), as the vector of Foreign factors employed in performing the offshored tasks, i.e. \( A_1^T X_o \). Let \( V_o = V + \Delta V \) and \( V_o^* = V_o - \Delta V \) denote the shadow-migration-adjusted endowments, where \( \Delta V = A_1^T X_o > 0 \). We may now rewrite the full-employment conditions of equation (6) in terms of shadow-migration-adjusted endowments,
\[ V_o = A^T X_o, \quad V_o^* = \gamma A^T X_o^*, \quad (*) \]

where \( \Delta V = A^T X_o > 0 \). Observe that these modified full-employment conditions are isomorphic to those of equation (1), which pertains to the equilibrium with free trade-in-goods but no trade-in-tasks.

Using the shadow-migration transformed employment conditions (9), Home’s import of final goods under trade-in-tasks, \( M_o \), is related to endowments by:

\[ M_o = \sigma X_o^w - X_o = (A^T)^{-1} (\tilde{V}_o^w - V_o) \quad (10) \]

where \( \tilde{V}_o^w = \tilde{V}^w + (1 - \gamma^{-1}) \Delta V \), namely, the effective world endowment rises (note that \( \tilde{V}_o^w > \tilde{V}^w \)) because a subset of Foreign workers use Home’s superior technology at the trade-in-tasks equilibrium. Inspection of this expression yields:

**Proposition 5 (trade-in-tasks analogue to HO theorem).** The pattern of trade in goods in the trade-in-tasks equilibrium is explained by the HO theorem where actual endowments are replaced by shadow-migration-adjusted endowments.

Such adjustments are symmetric to those introduced in Davis and Weinstein (2001) to account for non-tradable goods; here, the adjustment accounts for tradable tasks. This is subject to the usual provisos that apply to higher-dimension versions of the HO and HO theorems – see Ethier (1974, 1984) or Dixit and Norman (1980).

The equilibrium expression (10) in particular is amenable to empirical testing using ‘gross trade flows’ in final goods. Specifically, we could use the World Input-Output Database to calculate the imported intermediates in the final-good import vector of a nation. Then, combining this with factor intensity figures from the literature (e.g. from national IO tables), we could calculate \( (\sigma \tilde{V}_o^w - V_o) \). Finally, we could reproduce all the usual HO tests that have been undertaken on ‘native’ endowment vectors (e.g. Trefler

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15 Relaxing the assumption for Leontief technologies would require to include an additional adjustment to account for the fact that the technology matrix of Foreign, \( A_o^* \), is different from Home’s, \( A_o \), in the offshoring equilibrium. Without access to country-specific input-output matrices, then the only adjustment in (11) accounts for task imports.
1993, Davis and Weinstein 2001) using these shadow-migration-adjusted endowments instead.

Switching to the HO approach and using the definition of $\tilde{V}_0^w$:

$$\left( s\tilde{V}^w - V \right) - A^T M_o = \left[ 1 - s(1 - \gamma^{-1}) \right] \Delta V. \quad (11)$$

Without trade-in-tasks, the HO theorem asserts that the Home’s next exports of final goods, i.e. the left-hand-side of equation (11) above, should be zero as per equation (2). With trade-in-tasks, Home’s net exports of final goods must pay for its imports of tasks from Foreign, i.e. the right-hand-side of equation (11). In addition:

**Corollary 5.1:** The difference between the factor-content predicted by the HO theorem and the measured factor-content of Home’s import vector, $A^T M_o$, is (i) proportional to and smaller than the shadow migration vector $\Delta V$; (ii) this difference is increasing in the size of the country (ceteris paribus) and is exactly equal to $\Delta V$ in the limiting case of $s = 0$ (a country so small in the world that its size is negligible); and (iii) this difference is decreasing in the technology gap (ceteris paribus) and is exactly equal to $\Delta V$ in the limiting case of $\gamma = 1$ (technological differences are zero).

The proof is by inspection of (11). In the two-factor example (skilled and unskilled workers) where Home is skill-abundant and coordination costs are such that the offshored tasks are particularly unskilled intensive, Home’s shadow-migration-adjusted endowment is skewed towards unskilled labour, so, as per Proposition 4, it may import the skill-intensive good for reasons that are conceptually different from the exogenous Ricardian differences suggested by Leontief (1953) and confirmed by Trefler (1993). From (11),

$$\left( s\tilde{V}^w - V \right) - (A^T M_o + \Delta V) = -s(1 - \gamma^{-1})\Delta V \quad \text{and combining this with Corollary 5.1 we have:}$$

**Corollary 5.2 (bounded HO errors):** In the presence of trade-in-tasks, the standard HO factor-content prediction, $s\tilde{V}^w - V$, should overstate the factor-content of final-goods trade but understate the factor-content of final-goods trade plus that of trade-in-tasks. More precisely, the factor-content of final-goods and traded
tasks are $A^T M_o$ and $\Delta V$ respectively, so the following bounds should hold:

$$A^T M_o < s\tilde{V}^* - V < A^T M_o + \Delta V.$$  

The proof is by inspection of (11) noting that every element of $\Delta V$ is non-negative.

**Corollary 5.3 (source of comparative advantage):** Offshoring is a source of comparative advantage in the sense that trade-in-tasks creates trade-in-goods that would not occur otherwise.

That is, offshoring alters the pattern of trade, as per Proposition 5 or inspection of (6). Consider the special case where Home and Foreign have proportional factor endowments (i.e. $V = bV^*$, $b > 0$), so no trade occurs in the trade-in-goods equilibrium. Allowing trade-in-tasks creates Ricardian comparative advantage (unless the usual knife-edge case $A_i = \phi A$ arises) and thus trade in final goods. In addition, Home must export final goods to balance its imports of Foreign tasks.

In practice, task trade may be recorded as intraindustry trade because some final goods exports embody imports of components within the same industry classification at some level of aggregation. For instance, exports of Motor Vehicles (NAICS 3361) and imports of Motor vehicle parts (NAICS 3363) would account as intraindustry trade at the three-digit level of aggregation (NAISC 336 for both) but such trade is better recovered by the concept of ‘vertical specialisation of trade’ (Hummels, Ishii and Yi, 2001). Let $a^T_{i1}$ denote the row vector of offshored tasks in industry and $-m_{c0,i}$ denote exports of final goods in industry $i$, then Hummels et al.’s index of vertical specialisation of industry $i$ is

$$VS_i = \frac{a^T_{i1} w^*}{-P_{o,i} m_{c0,i}}$$

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16 Grubel-Lloyd and other indices quantifying intraindustry trade also suffer from a well-known aggregation bias: for instance, exports of Automobiles (NAICS 33611) and imports of Heavy trucks (NAICS 33612) would account as interindustry trade at the five-digit level of aggregation but as intraindustry trade at the four-digit level (NAISC 3361 for both).
if \(-m_{o,j}\) is positive and zero otherwise. This would be recorded as intraindustry trade if task/component import transactions, \(a_i^T w_{o}^*\), are recorded in the same industry \(i\) classification as final good exports, \(-p_{o,j} m_{o,j}\). Formally, we write:

**Corollary 5.4 (vertical specialisation and intraindustry trade):** If the offshored tasks produce intermediate goods then intraindustry trade arises.

We next turn to showing that going from no trade in tasks to trade in tasks is equivalent to particular changes in prices and endowments, whose impact can be studied using the (existing) Rybczynski and Stolper-Samuelson theorems.

Assuming that the set of goods being produced is constant and its number equal to the number of factors \(F\), analogues for the Stolper-Samuelson and Rybczynski theorems are straightforward. Inverting the Home pricing equation in (8), Home’s equilibrium factor prices are equal to \(w_o = A^{-1}(p_o + S)\) at the trade-in-tasks equilibrium while it is equal to \(w = A^{-1}p\) at the trade-in-goods equilibrium (without task trade); Foreign wages are only affected by terms-of-trade changes. Home’s shadow endowment is equal to \(V_o = A^T X_o\) by equation (9), so Home production is equal to \(X_o = (A^T)^{-1}(V + \Delta V)\) with trade-in-tasks while it is equal to \(X = (A^T)^{-1}V\) without task trade; analogous expressions hold for \(X_o^*\) and \(X^*\). With \(\Delta p \equiv p_o - p\), the equations of change are:

\[
\begin{align*}
X_o - X &= (A^T)^{-1}\Delta V, \\
w_o - w &= A^{-1}(\Delta p + S), \\
X_o^* - X^* &= -(\gamma A^T)^{-1}\Delta V, \\
w_o^* - w^* &= (\gamma A)^{-1}\Delta p
\end{align*}
\]

(12)

Under the assumption that the number of goods being produced is constant and equal to the number of factors, we may write:

**Proposition 6 (trade-in-tasks analogue to Stolper-Samuelson theorem).** Starting from free trade in goods, allowing trade-in-tasks affects Home factor prices in exactly the way predicted by the standard Stolper-Samuelson theorem with the vector of cost-savings from offshoring \(S\) coming in addition to the usual exogenous variation in prices.
Proposition 7 (trade-in-tasks analogue to Rybczynski theorem). Starting from free trade-in-goods, allowing trade-in-tasks affects production in exactly the way predicted by the standard Rybczynski theorem with the implied ‘shadow migration’ replacing the usual exogenous variation of factor endowments. Standard Jonesian magnification effects occur.

The proofs are by inspection of (12), noting that the production-change and the wage change problems have been reduced to the standard Rybczynski and Stolper-Samuelson theorem thought-experiments, so the impact on production is as predicted by the Rybczynski and Stolper-Samuelson theorems.

Note that the conditions under which the Stolper-Samuelson magnification effect arises are fairly general and, in particular, do not hinge on Assumption 3 (Leontief technologies). Generalising the conditions under which Proposition 6 applies only requires the absence of joint production and that each sector combine two primary factors or more to produce only one output (Jones and Scheinkman 1977). By contrast, generalizing Proposition 7 is problematic when there are more factors than goods. In the ‘even case’ $I = F$, each factor has at least one ‘natural enemy’, that is, the price of $f$ falls as the price of $i$ rises while all other prices remain constant. The dual of this proposition is that for any $f$, there exists an $i$ such that the output of $i$ falls as the endowment of $f$ rises, keeping good prices constant because $\partial w_i / \partial p_f = \partial x_i / \partial V_f$ holds for all $i$ and $f$ in this case (Jones and Scheinkman 1977, Dixit and Norman 1980).

Standard trade theory rarely addresses the impact of free trade on global output. With trade-in-tasks, however, there are important and systematic global changes in output since shadow migration expands the effective world endowments, i.e. $\tilde{V}_o > \tilde{V}^w$. From (12) and the definition of $X^w$ we get:

$$X^w_o - X^w = (1 - \gamma^{-1}) (A^T)^{-1} \Delta V.$$

Each factor having a natural enemy, it follows that shadow migration of a single factor reduces the production of at least one good if all factors are employed in equilibrium, that is shadow migration is like actual migration in this respect:

Proposition 8 (global production effects). If trade-in-tasks produces shadow migration in only one factor, then global production of at least one good must rise and that of at least one other good must fall.
The proof is by the usual Ethier (1984) approach to the I x F version of the Rybczynski theorem.\textsuperscript{17}

As a minor corollary, we note that the expansion of the shadow-migration-adjusted world endowment vector is proportional to the augmentation of Home’s shadow-migration-adjusted endowment, thus the global production effects tend to be proportional to Home’s production effects as shown by comparison of (12) and (13).

5. Integrating previous work in the literature

In this section, we use our framework to revisit previous work and well-known cases. We also illustrate how the various cases fit together. As most the literature works with what are effectively 2x2x2 models, we follow suit. Unlike several papers, we maintain Assumption 3 (Leontief technologies) throughout. This enables us to integrate the various papers into a unifying treatment.

5.1. The 2x2x2 case

The 2x2x2 version of the HO model is a key source of theoretical insights for trade-in-goods and a workhorse of the offshoring/trade-in-tasks literature. Jones and Scheinkman (1977) show how the results pertaining to the two-factor-by-two-sector trade models extend to higher dimensions in general. Here we impose \( I = F = 2 \) and we present the analytic solutions for the trade-in-tasks and the trade-in-goods equilibria.

The two factors (skilled labour \( K \) and unskilled labour \( L \)) are paid \( r \) and \( w \), respectively and work in the \( X \) and \( Y \) sectors. \( X \) is numeraire and \( L \)-intensive (i.e. \( \kappa_Y > \kappa_X \)) where \( \kappa_i \equiv a_{ki} / a_{Li} \) for \( i = X, Y \). Foreign is abundantly endowed with unskilled labour (i.e. \( k^* < k \) where \( k \equiv K / L \) and \( k^* \equiv K^* / L^* \)). To ensure diversified production with free trade in goods, we assume \( \kappa_Y > k > k^* > \kappa_X \) when the \( \kappa \)'s are evaluated at the equilibrium factor prices. Preferences are homothetic.

\textsuperscript{17} From the full-employment condition of the expanding factor, we know that the proportional expansion in the factor equals the average of the proportional changes in outputs weighted by employment-shares. From the employment condition for some non-expanding factor, we know that the employment-share-weighted average proportional changes in output must be zero. The only way both can be true is if at least one output expands and one contracts.
Inverting expressions in (1) yields solutions for $w$’s and $X$’s in the trade-in-goods equilibrium:

$$w = A^{-1}p, \quad w^* = \gamma^{-1}A^{-1}p, \quad X = (A^T)^{-1}V, \quad X^* = \gamma^{-1}(A^T)^{-1}V^*,$$

where

$$w \equiv \begin{bmatrix} w \\ r \end{bmatrix}, \quad p \equiv \begin{bmatrix} 1 \\ p \end{bmatrix}, \quad V \equiv \begin{bmatrix} L \\ K \end{bmatrix}, \quad X \equiv \begin{bmatrix} X \\ Y \end{bmatrix}, \quad A \equiv \begin{bmatrix} a_{lx} & a_{kx} \\ a_{ly} & a_{ky} \end{bmatrix}, \quad A_1 \equiv \begin{bmatrix} a_{lx1} & a_{kx1} \\ a_{ly1} & a_{ky1} \end{bmatrix}.$$  

Global market-clearing yields

$$p = \frac{\alpha}{1 - \alpha} \frac{a_{ly} \kappa_Y - \tilde{k}_w}{a_{lx} \kappa_X - \kappa_L},$$

where $\alpha \in (0,1)$ denotes the equilibrium expenditure share on $Y$ and

$$\tilde{k}_w \equiv \frac{K + K^* / \gamma}{L + L^* / \gamma}$$

is the world’s capital-to-labour relative endowment in effective units.

Consider next the trade-in-tasks equilibrium. Using (14) and solving (9) and (8) for $X_0$ and $w_0$ yields:

$$X_0 - X = \frac{1}{a_{ly}} \frac{\kappa_Y \Delta L - \Delta K}{\kappa_Y - \kappa_X}, \quad Y_0 - Y = \frac{1}{a_{ly}} \frac{\Delta K - \kappa_Y \Delta L}{\kappa_Y - \kappa_X},$$

$$w_0 - w = \frac{a_{kx} S_x - a_{kx} S_y}{a_{lx} a_{ly}(\kappa_Y - \kappa_X)} + A^{-1} \begin{bmatrix} 0 \\ \Delta p \end{bmatrix}, \quad r_0 - r = \frac{a_{ly} S_x - a_{lx} S_y}{a_{lx} a_{ly}(\kappa_Y - \kappa_X)} + A_{1}^{-1} \begin{bmatrix} 0 \\ \Delta p \end{bmatrix}$$

(15)

where $\Delta p = p_o - p$ is the terms-of-trade variation, $S_x = a_{lx1}(w_o - w_o^*) + a_{kx1}(r_o - r_o^*)$ and $S_y = a_{lx1}(w_o - w_o^*) + a_{kx1}(r_o - r_o^*)$ are the cost savings in sectors $X$ and $Y$, and $\Delta K = a_{kx1} X_o + a_{kx1} Y_o$ and $\Delta L = a_{lx1} X_o + a_{lx1} Y_o$ are the shadow migrations of $K$ and $L$ that arise with trade in tasks, respectively.\(^\text{18}\)

Figure 1: Necessary and sufficient conditions for wage and production effects due to trade in tasks.

\(^{18}\) For example, $\Delta V/X = [(\Delta L/L) / (1 - k/k\gamma) - (\Delta K/K)/(\kappa_Y / k - l)]$ and $k/k\gamma < 1.$
Expression (15) shows the necessary and sufficient conditions for signing production and wage effects of trade-in-tasks. Rather than write out the results in the form of propositions, we depicted the full range of outcomes in Figure 1. For example, if shadow migration is heavily skewed towards $L$ (specifically, $\Delta K/\Delta L$ is lower than the capital intensity of the $L$-intensive good $X$, $\kappa_X$) then $X$ rises and $Y$ falls. If shadow migration has an intermediate factor ratio, namely $\kappa_X < \Delta K/\Delta L < \kappa_Y$, then both $X$ and $Y$ rise. Finally, if it is heavily skewed towards $K$ ($\Delta K/\Delta L > \kappa_Y$), then $X$ falls and $Y$ rises.

Foreign production effects are characterised in an isomorphic manner.

Turning to the wage effects, we see from (15) that the wage of Home $L$-workers rises (controlling for terms-of-trade effects) if and only if the cost-saving is sufficiently greater in the $L$-intensive sector than in the $K$-intensive sector, namely, $w_0 > w \iff S_X / S_Y > a_{XX} / a_{KY}$; in this case, $r$ may actually fall. The necessary and sufficient condition for $r$ to fall (controlling for terms-of-trade effects) is that the ratio of cost-savings exceeds the ratio of $L$-input coefficients, namely, $S_X / S_Y > a_{LY} / a_{LY}$. If the cost-savings ratio lays between the skilled-unskilled endowment ratios, then both $w$ and $r$ rise by an effect analogous to product augmenting technical change. Controlling for the terms-of-trade effects, trade-in-tasks has no effect on foreign wages because Foreign firms do not benefit from this technical change.

**5.2. Integrating specific cases in the literature**

Most of the literature ignores terms-of-trade effects by imposing the small open economy assumption. In this case, the impact of offshoring on $w$ and $r$ are given by the bottom row of (15) taking $\Delta p = 0$.

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19 The appendix of our working paper provides exact necessary and sufficient conditions for diversification in the 2x2x2 version of the model (http://www.dagliano.unimi.it/media/wp2008_250.pdf).
**Offshoring as sector-specific technical progress.** Many papers assume that offshoring occurs in only a single sector. This includes Jones and Kierzkowski (1990) and follow-on papers,\(^{20}\) Deardorff (1998a), Venables (1999), Kohler (2004a, b), and Markusen (2006). In such papers, either \(S_X = 0\) or \(S_Y = 0\), so offshoring acts like sector-specific technical progress. The wage effects thus depend on the factor intensity of the progressing sector; offshoring in the \(L\)-intensive sector only, namely \(X\), raises \(w\) and lowers \(r\); while offshoring in \(Y\) only does the opposite, as (15) shows. That is to say, offshoring in unskilled labour intensive sectors raises unskilled wages – even if the tasks being offshored are themselves labour intensive.

**Offshoring as factor-specific technical progress.** Other papers consider offshoring involving only one factor but in both sectors, so offshoring is like factor-specific technical progress, specifically \(S_X = a_{LX}(w_o - w_o^*)\) and \(S_Y = a_{LY}(w_o - w_o^*)\). Factor-specific technical progress has ambiguous effects on \(w\) and \(r\) (Jones 1965); what matters is the relative size of the cost savings by sector (the necessary and sufficient conditions are summarised by the left-panel in Figure 1). That is to say, offshoring of unskilled tasks may raise unskilled wages because such trade in tasks tends to reduce production costs in unskilled labour intensive sectors more than in skilled labour intensive sectors.

This result was famously popularized in the main body of analysis of GRH: unskilled labour unambiguously gains from the offshoring of unskilled-intensive tasks while the other factor’s wage effect is exactly zero. How does this fit in with the ambiguity apparent in (15)? This sharp result is driven by the concatenation of three assumptions. Working in what could be boiled down to a 2x2x2 model, GRH describe the production process of each sector as involving two continuums of tasks – one that uses only \(L\), the other only \(K\). The four continuums are normalised to the unit interval, and within each continuum, tasks are assumed to use the same amount of the relevant factor. After ordering the tasks by increasing offshoring costs, GRH normalise the offshoring costs across sectors. In the famous special case, only \(L\)-tasks are offshoreable, but the two normalisations and the assumption of identical task-intensity

\(^{20}\) For example, Jones and Marjit (1992), Arndt (1997, 1999), Jones and Kierzkowski (1998, 2000), and Jones, Kierzkowski and Leonard (2002). Francois (1990a, b, c) explicitly considers the impact of offshoring on the factor price equalization set.
imply that exactly the same fraction $\lambda$ of $L$ is offshored in $X$ and $Y$ at equilibrium. In our notations $S_X = \lambda L_X (w_O - w^*_O)$ and $S_Y = \lambda L_Y (w_O - w^*_O)$. Plugging these into (15) yields $r_O - r = 0$ and $w_O - w = \lambda (w_O - w^*_O)$. Ambiguity of the wage effects is restored in subsequent analysis in GRH when they drop the constant proportion assumption or allow for offshoring of $H$-tasks.

6. Extending the basic model

We extend our integrating model to allow technology diffusion, Ricardian differences among nations that result in two-way offshoring – a common phenomenon among OECD nations (Amiti and Wei 2005) – and to incorporate monopolistic competition following Helpman and Krugman (1985).

6.1. Offshoring with technology diffusion

In the previous section, all output of the offshored sector was ‘sold’ to Home. Here we allow local sales of offshored tasks; put differently, we assume that Home technology diffuses to Foreign for the offshored tasks. This version of the model captures well long-run technology spillovers brought about by vertical FDI (Javorcik 2004) and it is also consistent with e.g. Apple’s offshore outsourcing activities.

Home firms have an incentive to sell type-1 tasks to Foreign producers as their superior technology gives them an edge over local producers. Imposing Assumption 3 (Leontief technologies) in this section to reduce clutter, the pricing and employment equations with local sales of offshored tasks are:

$$p_o + S = Aw_o, \quad p_o + S^* = \gamma Aw^*_o; \quad V + \Delta V = A^T X_o, \quad V^* + \Delta V^* = \gamma A^T X^*_o. \quad (16)$$

First, we solve (16) and (1) for wages; this yields:

$$w_o - w = A^{-1}(S + \Delta p), \quad w^*_o - w^* = A^{-1}(S^* + \Delta p), \quad (17)$$

where $\Delta p \equiv p_o - p$ and $S \equiv A_1 (w_o - w^*_o)$ are as in Section 4. In addition, Foreigners now benefit directly from offshoring’s cost saving, whereas they were only indirect affected via terms-of-trade effects in the case of Sections 3 to 5.

There is a crucial difference, though, between the factor price effects on Home versus Foreign labour. For Home labour, it is rents that generate the cost-savings (i.e. the fact that Home firms use their superior technology but pay Foreign wages); for Foreign
labour it is technology diffusion that generates the cost-savings and this cost-savings are proportional to the technology gap: \( S^* = (\gamma - 1)A_1w^*_0 > 0 \). Despite this qualitative difference, the Foreign wage changes in (17) are isomorphic to those of Home. Consequently, all the detailed analysis in the previous sections relating the cost-savings to the wage effects is now applicable to the impact of offshoring on Foreign wages, too.

Second, we solve (16) for production effects; using (14) yields:

\[
X_0 - X = (A^T)^{-1}\Delta V, \quad X^*_0 - X^* = \gamma^{-1}(A^T)^{-1}\Delta V^*.
\]

Three aspects of this expression are noteworthy. First, \( \Delta V = A^T_1X_0 \) is as in Section 4. By contrast, the impact on Foreign production is qualitatively different because \( \Delta V^* = -\Delta V + (\gamma - 1)A^T_1X^*_0 \). Second, the shadow migration interpretation is less clear-cut than in Section 4: the first term in the right-hand side of the definition of \( \Delta V^* \) does relate to the shadow migration of Foreign workers conveying Home tasks and this effect tends to reduce Foreign production of final goods, but the second term, \( (\gamma - 1)A^T_1X^*_0 \), relates to the technology diffusion that now benefit all Foreign workers conveying offshoreable tasks. This increase in effective factor endowments is proportional to the technology gap and it tends to expand Foreign production. The net effect on Foreign production of final goods is thus ambiguous. Finally, effective world endowments are unambiguously larger with offshoring, i.e. \( \tilde{V}^*_0 > \tilde{V}^* \). In the case without technology diffusion (Sections 3 to 5), Home offshores technology that is used only for Home production, so the Foreign labour employed in the offshoring sector is diverted from Foreign production and this means that the Foreign production change is proportional to the Home production effect but of the opposite sign. Here the tech-transfer embodied in offshoring diffuses to and stimulates Foreign production and this simple proportionality breaks down. Nevertheless, the basic analysis of production effects for Foreign follows the reasoning of Proposition 4 and Figure 1 with \( \Delta X^* \) substituted for \( \Delta X \).

Since the trade effects follow from the production and factor price changes, it is clear that offshoring in technology-diffusion case at hand will also be a source of comparative advantage and intra-industry trade.
To summarise, the main difference between the two cases is that offshoring with technology diffusion spreads some of the benefit of the implicit technology transfers to Foreign factors whereas in the case all studied in Sections 3 to 5 the benefits accrued to Home factors (modulo terms of trade effects).

6.2. Ricardian differences and intra-industry two-way offshoring

Davis (1995) shows that intraindustry trade arises in a HO-like model due to minor technological differences among nations when there are more goods than factors. As many production patterns are consistent with the system of equations in (1) when \( I > F \), even minor technological advantages can shift global production of individual goods to a single nation. We apply this insight to generate two-way trade-in-tasks that arises from task-level technology differences across nations (e.g. Italy may be especially excellent at making brakes for small cars, while France may be especially excellent at making air bags for small cars, even though France and Italy are roughly at parity when it comes to small car technology).

To implement this idea cleanly, we eliminate all macro differences between Home and Foreign by assuming \( \gamma = 1 \), and \( V = V^* \) (we do not impose Assumption 3). The trade-in-goods equilibrium is thus marked by absolute FPE and zero trade. There are, however, task-level technology differences in the sense that Foreign’s task technology is as in (3), but Home’s task technology is now:

\[
I_iF_i\alpha_i(w) = \sum_{t=1}^{N_t} \epsilon_{it} a_{fi}(w); \quad \forall f = 1, \ldots, F; \quad i = 1, \ldots, I
\]

where \( \epsilon \in [1-\mu, 1+\mu] \) is a random variable that is iid across sectors and tasks, symmetrically distributed around \( E\{\epsilon\} = 1 \) and with \( \mu > 0 \).\(^{21}\)

We assume that all tasks are potentially offshorable and firms can supply tasks to one another. We also assume that \( N_t \) is sufficiently large for all industries and that the coefficients \( a_{fi} \) are symmetrically distributed around \( a_i \), so that the realization of \( a_{fi}(\cdot) \) in Home is arbitrarily close to \( a_{fi}^*(\cdot) \) in Foreign; thus, factor prices are equalised and Home is competitive in all tasks where \( \epsilon_{it} < 1 \) while Foreign has the edge in all other tasks. The law of large numbers implies that Home has the edge in half the tasks.

\(^{21}\) Here we treat the realizations of \( \epsilon \) as parameters. These are the endogenous result of external scale economies in the recent work of Grossman and Rossi-Hansberg (2012).
sector-by-sector. Moreover, the tasks in which Home has the Ricardian comparative advantage will be a random sample of all tasks, so the Home employment condition will be:

\[ V = \frac{1}{2} A^T X + \frac{1}{2} A^T X^*. \]

As Home and Foreign are symmetric at the macro level, it is clear that trade-in-tasks will have no impact on the \( w \)'s or \( X \)'s, but intraindustry offshoring and intraindustry trade-in-tasks will arise. There are no terms-of-trade effects, so gains from trade-in-tasks are assured. To see this, note that \( A_0^T = \frac{1}{2} \left[ 1 + F_{\varepsilon} (1) \right] A^T = \frac{1}{2} A^T < A^T \) holds (by the law of large numbers), where \( F_{\varepsilon}(\cdot) \) is the CDF of \( \varepsilon \). Further, all factor owners are better off if preferences are homothetic; to see this, note that \( w_o > w \) follows by unit cost pricing (\( w_o = A_0^{-1} p_o \) and \( w = A^{-1} p \)) and homothetic preferences imply \( p_o = p \).

### 6.3. Offshoring in a Helpman-Krugman trade model

A fact that has been well appreciated in trade theory since Helpman and Krugman (1985) is that the basic HO results carry through unaltered in a Dixit-Stiglitz monopolistic competition setting provided that the increasing returns technology is homothetic, i.e. the cost function is equal to \((mx + f) \sum_f a_f w_f\), where \( m \) and \( f \) are parameters that govern marginal and fixed costs, respectively, and \( x \) is firm-level output. Here we use this insight to show that the above analysis could easily be conducted in a monopolistic competition trade model setting.

The key to the Section-3 analysis lies in the pricing and employment equations and their restatement using the shadow migration insight. As is well known, the free-entry output of a typical variety under monopolistic competition with homothetic technologies depends only on cost and taste parameters and so does not vary across the equilibria we consider. This implies that monopolistic competition sectors display constant returns at the sector level (doubling all inputs results in doubling sectoral output at equilibrium), specifically, \( X = n\bar{x} \) is the sector’s total output where \( \bar{x} \) is the invariant firm-level output. The sector’s employment of factor \( f \) is thus \( nx(m + f / \bar{x})a_f \) where \( i \) is the Dixit-Stiglitz sector. Likewise the price of the Dixit-Stiglitz sector equals average cost, namely \((m + f / \bar{x}) \sum_f a_f w_f\). Choosing units such
that $m + f / \bar{x}$ is equal to unity, the employment and pricing equations for this model are identical to those of the HO model of Section 3, as are the Foreign pricing and employment conditions. With this, we have reduced the problem to the one solved in Sections 3, so can conclude that the relevant Propositions also in a model that allows monopolistic competition.

7. Summary and concluding remarks

Recent theoretical contributions have renewed interest in characterising the effects of offshoring and the result has been a wide range of cases that generate unexpected results – many which seem to contradict intuition based on standard trade theory. This paper is an attempt to integrate the theoretical trade-in-tasks literature into standard trade-in-goods theory. We present a simple modification of the HO model that allows us to consider trade-in-goods in the traditional sense (i.e., the exogenous shift from no-trade to free-trade in goods) as well as trade-in-tasks (i.e. the exogenous shift from no-trade to free-trade in a range of routine tasks).

The expressions for the trade and production patterns, and goods and factor prices are highly complex in the trade-in-tasks equilibrium and clearly violate the standard HO predictions. However, if one views offshoring as ‘shadow migration’, and uses shadow-migration adjusted endowments instead of actual endowments, the HO trade and production predictions work perfectly. As such, we can use the elegant HO theorems to establish necessary and sufficient conditions for the trade and production effects of offshoring. As the quantity of factors employed in offshore production is potentially observable with firm-level datasets, these trade-in-tasks analogues of the HO and Rybczynski theorems are testable in principle. We also show that offshoring creates intra-industry trade when the various tasks are considered as being in the same sector. On the price side, we show how using the vector of the cost-savings that ‘shadow migration’ produces can be used to develop trade-in-tasks analogues of the FPE and Stolper-Samuelson theorems.

Our integrating framework does not encompass the many important contributions in the literature that focus on issues of corporate governance, e.g. Antràs and Helpman (2004). These papers typically focus on the division of rents and how they depend upon the corporate structure chosen. As the division of rents will affect the division of the benefits from offshoring, we conjecture that it could have significant general
equilibrium effects as well as the more direct effects on ownership. Incorporating such issues would seem to be an important topic for future theoretical research.

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References


Deardorff, A. V. (1994)


