Dispersion economics

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Abstract

The cost of living in cities is higher as a result of crowding and commuting.

1 Crowding: the Helpman model

The set up is as follows:

- People are freely mobile: reservation real wage ω_0.
- Exogenous stock of city c housing area: H_c.
- Unit price of housing square meter: r_c.
- Labour income: w_c.
- Endogenous city population: L_c.
- Preferences: $U = h^\alpha y^{1-\alpha}$, where h is per-capita consumption of housing (thus $h_c = H_c/L_c$) and y is consumption of a freely traded numéraire good (i.e. its price is unity: $p_y = 1$).

1.1 Equilibrium

With Cobb-Douglas preferences, a constant fraction α of consumer spending is devoted to housing. Expenditure is equal to labour income so $r_c h_c = \alpha w_c$. Together, these imply

$$r_c = \alpha w_c \frac{L_c}{H_c}.$$

The real wage (or indirect utility) is equal to

$$\omega_c \equiv \frac{w_c}{p_y^{1-\alpha} r_c^\alpha} = w_c r_c^{-\alpha} = w_c^{1-\alpha} \left(\frac{H_c}{\alpha L_c} \right)^\alpha .$$

Spatial equilibrium. All cities that have a positive population must provide the same real wage (or indirect utility):

$$w_c = \omega_0^{1/(1-\alpha)} \left(\frac{\alpha}{H_c}\right)^{\alpha/(1-\alpha)} L^{\alpha/(1-\alpha)}.$$

For $H_c = H$, all c, some $H > 0$, we may rewrite this expression as

$$\ln w_c = a + \gamma \ln L_c,$$

with $\gamma \equiv \alpha/(1-\alpha)$ and $a \equiv [\ln \omega_0 + \alpha(\ln \alpha - \ln H)]/(1-\alpha)$. Alternatively, for $H_c = HL_c^{\phi}$, some $H > 0$ and some $\phi \in (0,1)$, we may rewrite the wage equation as

$$\ln w_c = a + \gamma \ln L_c,$$

with $\gamma \equiv (1-\phi)\alpha/(1-\alpha)$ and a is defined as before.

Either way, wages are increasing in city size.

1.2 Dispersion economies

Wages are increasing in city size in this model because housing prices are increasing in city size as a result of crowding: indeed, population density,

$$d_c \equiv \frac{L_c}{H_c},$$

is increasing in L_c in both cases. Mobile people accept to pay higher rents or housing prices only if they are properly compensated with a higher wage.

2 Congestion: the Alonso-Mills model

- People are freely mobile: reservation real wage ω_0.
- Cities can expand horizontal along one dimension. Endogenous stock of city c housing area: H_c.
- Each individual consumes exactly one unit of land for housing so that $H_c = L_c$.
- Labour income earned in CBD (central business district): w_c.
- Workers commute between the CBD and their home. Per-unit distance commuting cost: $\tau h^{\gamma} > 0$, where h here denotes commuting distance. τ and γ are positive parameters.
- Endogenous city population: L_c.
- Preferences: people consume one unit of housing, pay for commuting and spend the rest of their income on the numéraire good y (thus $p_y = 1$). Income includes labour income and a fair share of land rents.

2.1 Equilibrium

At a spatial equilibrium within the city, everybody must be indifferent between living close to the CBD and far from it. Thus

$$R(h) + \tau h^{\gamma} = R_0,$$

where

$$R_0 \equiv R(0)$$

is the rent paid by people living by the CBD (these people's commuting cost is zero). The longest commute is $L/2$ and people at the fringe pay an exogenous rent R_A (which is the rent paid by the best alternative use, e.g. agriculture). Income net of rents and commuting costs has to be the same everywhere at a within-city spatial equilibrium so that

$$R_0 = R\left(\frac{L}{2}\right) + \tau\left(\frac{L}{2}\right)^{\gamma} = R_A + \tau\left(\frac{L_c}{2}\right)^{\gamma}$$

and, more generally,

$$R(h) = R_0 - \tau h^{\gamma} = R_A + \tau\left[\left(\frac{L}{2}\right)^{\gamma} - h^{\gamma}\right]$$

so that the sum of rents and commuting cost (which is the same for every urban dweller) is

$$R(h) + \tau h^{\gamma} = R_0.$$

Total rents are

$$\begin{aligned} TR &\equiv 2\int_0^{L/2} R(h)\,\mathrm{d}h \\ &= LR_0 - \frac{2\tau}{1+\gamma}\left(\frac{L}{2}\right)^{1+\gamma} \\ &= \frac{2\gamma}{1+\gamma}\tau\left(\frac{L}{2}\right)^{1+\gamma} + LR_A. \end{aligned}$$

Average rents are

$$AR \equiv \frac{TR}{L} = \frac{\gamma}{1+\gamma}\tau\left(\frac{L}{2}\right)^{\gamma} + R_A.$$

Where do rents go? Two alternative modelling assumptions are possible. In one, rents go to so-called *absentee landlords*. In the other, rents are entirely redistributed to the local population. The latter is a better assumption in a general equilibrium of cities but, to accomodate both as extreme special cases, let s denote the share of land rents that is redistributed to the local population and $1 - s$ the share that is paid to absentee landlords.

Thus real wages are (reintroducing city-specific subscripts)

$$
\begin{aligned}
\omega_c &= w_c - R_{0c} + sAR_c \\
&= w_c - \left(1 - s\frac{\gamma}{1+\gamma}\right)\tau\left(\frac{L_c}{2}\right)^{\gamma} - (1-s)\,R_{Ac} \\
&= w_c - \frac{1+(1-s)\,\gamma}{1+\gamma}\tau\left(\frac{L}{2}\right)^{\gamma} - (1-s)\,R_{Ac} \\
&= w_c - (1-s)\,R_{Ac} - \theta L^{\gamma},
\end{aligned}
$$

where $\theta \equiv 2^{-\gamma}\tau\,[1+(1-s)\,\gamma]\,(1+\gamma)^{-1} > 0$. Note that real wages ω_c are unaffected by R_{Ac} when $s = 1$. [Why?]

Spatial equilibrium. All cities that have a positive population must provide the same real wage (or indirect utility), thus $\omega_c = \omega_0$ for all non-empty cities. Setting $R_{Ac} = R_A = 0$, all c, and the outside option to zero (i.e. $\omega_0 = 0$), this implies

$$\ln w_c = \ln\theta + \gamma\ln L_c.$$

In plain English, wages are increasing in city size.

2.2 Dispersion economies

Wages are increasing in city size in this model because rents and commuting costs (or congestion costs for short) are increasing in city size. Mobile people accept to cope with higher congestion costs only if they are properly compensated with a higher wage.