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Agglomeration and welfare: The core–periphery model in the light of Bentham, Kaldor, and Rawls

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Abstract

The objective of this paper is to apply different welfare approaches to the canonical model developed by Krugman, with the aim of comparing the only two possible market outcomes, i.e. agglomeration and dispersion. More precisely, we use the potential Pareto improvement criteria, as well as the utilitarian and Rawlsian welfare functions. No clear answer emerges for the following two reasons: (i) in general, there is indetermination when compensation schemes are used and (ii) the best outcome heavily depends on societal values regarding inequalities across individuals. However, simulations undertaken for plausible values of the main parameters suggest that there might be excessive agglomeration.

JEL classification: F12; R13
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1. Introduction

Recent developments in economic geography have focused on the reasons explaining the formation of economic agglomerations.[1] However, they have put aside the question of the social desirability of such agglomerations. Some authors have even expressed some reticence in doing such an analysis.[2] Hence, whether there is too much or too little agglomeration is still unclear, even in the canonical model developed by Krugman. Yet, speculation on this issue has never been in short supply and it is fair to say that this is one of the main questions that decision makers would like to address. There are several good reasons to believe that the market outcome is not efficient. Indeed, besides the standard inefficiencies generated by firms pricing above marginal costs, economic geography models contain new sources of inefficiency whose origin lays in the mobility of agents. Typically, firms and workers move without taking into account the benefits and losses they bring about to the agents residing in their new region, nor the benefits or losses they impose on those left behind. Accordingly, there is a priori no general indication as to the social desirability of agglomeration – the concentration of mobile activities in one region – or dispersion – an even distribution of these activities across regions. This is why, in this paper, we use different tools borrowed from welfare economics to evaluate the relative merits of these two configurations. Conversely, we show how such tools may be applied to an issue that has generated heated debates, i.e. the social implications of economic integration.

The objective of this paper is to use Krugman's core–periphery model with the aim of identifying conditions under which agglomeration or dispersion is the better social outcome (hence, we do not conduct a first best analysis).[3] In order to compare the two market outcomes – agglomeration and dispersion – that most economic geography models yield, we will use the traditional tools of public economics, starting from the least controversial criterion to various social welfare functions. Let us make it clear that the answer to the question above is not an easy one. Even though the setting provided by Krugman involves no technological externalities at all, its welfare analysis does not deliver a simple and unambiguous message. This is because the pecuniary externalities generated by the mobility of agents matter for welfare as the core–periphery model assumes monopolistic competition.

We show the following results. First, none of the two allocations Pareto dominates the other: the workers living in the periphery always prefer dispersion, whereas all those living in the core always prefer agglomeration. This is hardly a surprising result, given the terminology (core and periphery) used to describe the regions in the agglomerated

[1] This has been accomplished by combining monopolistic competition, increasing returns at the plant level, mobile and immobile labor, and transport costs within a general equilibrium framework of the Dixit–Stiglitz genre. See Krugman (1991), Fujita et al. (1999), and Baldwin et al. (2003) for more details.

[2] As observed by Anas (2001), this is rather surprising because the Dixit–Stiglitz model of monopolistic competition, which is the main building-block of economic geography, was itself developed to deal with a welfare question: how does a market characterized by increasing returns and product differentiation perform from the social point of view (Dixit and Stiglitz, 1977)?

[3] Other equilibria may exist but they are unstable (Robert-Nicoud, 2005).

configuration. We then turn to the *compensation mechanisms* put forward by Kaldor (1939) and Hicks (1940) to evaluate the social desirability of a move, using market prices and equilibrium wages to evaluate the compensations to be paid either by those who gain from the move (Kaldor), or by those who are hurt by the move (Hicks). This is not, however, the end of the story. As observed by Little (1949), it is not sufficient to look at the simple desirability of such compensations. We must also consider their feasibility at the corresponding equilibrium prices and wages. In other words, we must check that the compensations may be effectively paid and that the material balances conditions are met at the incomes earned by the workers after compensation. This can be achieved only within a general equilibrium framework, but this is precisely one of the main desirable features of the core–periphery model.

Given this proviso, we will show that, *provided that transport costs are sufficiently low, agglomeration is preferred to dispersion* in the following sense: all workers in the core can compensate those staying in the periphery, whereas those staying in the periphery are unable to compensate the workers who choose to move in what becomes the core. In addition, we do not just show the existence of transfers allowing for a Pareto-dominant move: we determine the precise value of these transfers. *Outside that range, we face a problem of indetermination* in the sense of Scitovsky (1941): none of the two configurations is preferred to the other with respect to both the foregoing criteria. At first sight, this seems to be a surprising result. In our opinion, such an indetermination may be considered as the "synthesis" of the very contrasted views that prevail in a domain in which the two tenets have many good reasons to be right. To our knowledge, this is the first time that such a welfare analysis is performed in a spatial and general equilibrium context. We believe that this approach, which does not involve any interpersonal comparison and rests on market prices and incomes determined in a general equilibrium context, is superior to many others.

The partial indetermination mentioned in the foregoing leads us to focus on various social welfare functions. Even though utilitarianism may lead to awkward results in spatial models (Mirrlees, 1972; Wildasin, 1986), we follow a well-established tradition in public economics (see, e.g. Atkinson and Stiglitz, 1980) and consider the CES family of social welfare functions, which encapsulate different attitudes toward inequality across individuals and includes the utilitarian and Rawlsian criteria as polar cases. As expected, *the relative merits of agglomeration then critically depend on societal values*. If society does not care much about inequality across individuals, we show that agglomeration (resp., dispersion) is socially desirable once transport costs are below (resp., above) some threshold, the value of which depends on the fundamental parameters of the economy.[4] In particular, the domain of transport cost values for which agglomeration is preferred vanishes completely when the social welfare function is Rawlsian. Even though these results are derived from social preferences defined on individualistic utilities, they lead to policy recommendations that may be regarded as being region-based. This is due to the

[4] This is reminiscent of Ottaviano and Thisse (2002) who show that there is a whole range of transport costs for which agglomeration is the only stable equilibrium, whereas dispersion is socially desirable. Nevertheless, unlike Krugman's, their model does not allow for income effects. The similarity between the two results is therefore worth stressing.

fact that, according to the core–periphery model, the market yields very contrasted distributions of workers and income.

The remainder of the paper is organized as follows. The model is presented in Section 2. Using independent normalizations that entail no loss of generality, we establish several results leading to simple expressions, which are used in the welfare analysis of the agglomerated and dispersed configurations. In particular, we show that the nominal wage of the mobile workers is the same, whatever the prevailing equilibrium configuration. The Pareto, Kaldor, and Hicks criteria are then studied in Section 3, whereas the CES family of social welfare functions is used in Section 4 to compare the two configurations. Section 5 concludes.

2. The model and some preliminary results

2.1. Assumptions and notation

Consider an economy with two regions, labelled 1 and 2, two sectors, the agricultural and the manufacturing sectors, and two types of labor, the mobile and the immobile, which we refer to as *skilled* and *unskilled*, respectively. The skilled only work in the manufacturing sector and the unskilled only in agriculture. This economy is endowed with L units of unskilled labor and H units of skilled labor. Because we assume below that individual preferences are homothetic, we may refer to a unit of labor as a "worker". However, in the sections of the paper dealing with welfare, we will recognize that individuals may be embodied with different numbers of labor units according to their type of labor. The spatial distribution of unskilled labor is fixed and uniform (that is, there are $L/2$ unskilled workers in each region), whereas the spatial allocation of skilled labor is endogenous. The agricultural sector produces a homogenous product under perfect competition and constant returns, using one unit of unskilled labor to produce one unit of output. This good is costlessly tradable, thus implying that the wage of the unskilled workers is the same across regions. The agricultural good is chosen as the numéraire so that $p_a=w_a=1$. The manufacturing sector produces a continuum of varieties of mass n of a horizontally differentiated good under monopolistic competition and increasing returns, using skilled labor as the only input. Any variety of this good can be shipped from one region to the other according to an iceberg technology described by the parameter $\tau>1$, meaning that τ units must be sent from the region of origin for 1 unit to be available in the region of destination, as suggested by Samuelson (1954).

Preferences are identical across all workers. Each individual has Cobb–Douglas preferences over the traditional good and an aggregate of varieties Q of the manufactured good, with an expenditure share μ and a constant elasticity of substitution across varieties $\sigma>1$. Formally, tastes are described by the following utility function:

$$u = A\left(\frac{Q}{\mu}\right)^{\mu}\left(\frac{z}{1-\mu}\right)^{1-\mu} \quad Q \equiv \left(\int_{i=0}^{n} q(i)^{(\sigma-1)/\sigma}\mathrm{d}i\right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $q(i)$ and z respectively represent the quantity consumed of the manufactured variety i and of the agricultural good by the representative individual and A is a scale parameter. If y denotes a consumer's income, her demand for the agricultural good is $z=(1-\mu)y$ and her demand for the composite good is $Q=\mu y/P$ where

$$P \equiv \left[\int_0^n p(i)^{-(\sigma-1)} \mathrm{d}i \right]^{-1/(\sigma-1)}$$

is the minimum expense needed to consume Q (Dixit and Stiglitz, 1977). Introducing these demand functions in the utility (1) yields the indirect utility function

$$V = AyP^{-\mu} \tag{2}$$

Finally, for the sake of completeness, we may write the individual demand for variety i as follows:

$$q(i) = \left[\frac{p(i)}{P} \right]^{-\sigma} Q.$$

Let us now account for the spatial distribution of consumers and producers as well as for transport costs. The demand of a worker located in region r=1, 2 for the agricultural good is $z_r=(1-\mu)y_r$ irrespective of the region in which the good is produced since trading the agricultural product is costless. The demand q_{sr} in region r for a variety produced in region s=1, 2 is as follows:

$$q_{sr} = p_{sr}^{-\sigma} \frac{Q_r}{P_r^{-\sigma}}$$

where p_{sr} is the common consumer price – i.e. inclusive of transport costs – of a variety produced in s and sold in r, so that the price index in region r is

$$P_r = \left(n_r p_{rr}^{1-\sigma} + n_s p_{sr}^{1-\sigma} \right)^{1/(1-\sigma)} \quad r = 1, 2 \text{ and } s \neq r$$

with n_r (resp., n_s) being the number of varieties produced in region r (resp., $s \neq r$) and $n_r+n_s=n$.

The market demand for variety i produced in region r is given by

$$D_r(i) = \mu \left(p_{rr}^{-\sigma} P_r^{\sigma-1} Y_r + p_{rs}^{-\sigma} \tau P_s^{\sigma-1} Y_s \right)$$

where Y_r is the income of region r and $\tau>1$ units of variety i must be shipped from r to $s \neq r$ for 1 unit to be available in s.

Turning to the supply side in region r, the production of the quantity q_r of any variety requires h_r units of skilled labor given by:

$$h_r = \alpha + \beta q_r \quad r = 1, 2 \tag{3}$$

where $\alpha>0$ and $\beta>0$ are respectively the fixed and the marginal labor requirements. Hence, production of any variety exhibits increasing returns to scale internal to the firm. The profit function of each firm located in region r is therefore $\pi_r = p_r q_r - w_r(\alpha + \beta q_r)$, where p_r and w_r are respectively the producer price and the wage rate of skilled labor prevailing in region r.

Without loss of generality, we may choose the unit of the manufactured good such that $\alpha = 1/\sigma$. Following Baldwin et al. (2003), we may also use the fact that the number of firms is continuous and choose the unit of the real line along which this number is measured for the following equality $\beta = (\sigma - 1)/\sigma$ to hold.[5] Each firm chooses its price, taking into account the market demand for its variety and taking the price index as fixed because the firm is negligible to the market. Applying the first order condition then yields:[6]

$$p_r = \frac{\sigma}{\sigma - 1}\beta w_r = w_r \quad r = 1, 2 \tag{4}$$

Because each firm faces an iso-elastic demand and transport costs are of the iceberg-type, mill-pricing is an equilibrium feature of the model. As a result, we have $p_{rr} = p_r$ and $p_{rs} = p_r \tau$ so that the price index becomes

$$P_r = \left[n_r p_r^{1-\sigma} + n_s(\tau p_s)^{1-\sigma}\right]^{1/(1-\sigma)} \qquad r = 1, 2 \text{ and } s \neq r \tag{5}$$

and the market demand for variety i may be rewritten as follows:

$$D_r(i) = \mu\left(P_r^{\sigma-1} Y_r + \tau^{1-\sigma} P_s^{\sigma-1} Y_s\right) p_r^{-\sigma}. \tag{6}$$

In each region, nominal wages adjust so that no firms want to enter or exit the market, that is, profits are zero. Because $\pi_r = [q_r(1-\beta) - \alpha]w_r = (q_r - 1)w_r/\sigma$, imposing $\pi_r = 0$ implies that the equilibrium output of any active firm is

$$q_r = 1 \quad r = 1, 2. \tag{7}$$

Since profits are zero in equilibrium, the income in region 1 (resp., 2) is given by:

$$Y_1 = \frac{L}{2} + w_1 \lambda H \quad Y_2 = \frac{L}{2} + w_2(1 - \lambda)H$$

where λ is the endogenous share of skilled workers located in region 1.

The distribution of skilled workers λ is governed by the difference of individual welfare between the two regions, the welfare of a skilled worker in region r being evaluated by her indirect utility function given by

$$V_r = A w_r P_r^{-\mu} \quad r = 1, 2 \tag{8}$$

Hence, the mobility of the skilled (and of firms) is determined by the real wage differential. All things being equal, the price index is lower in the region with the larger number of firms, thus implying that the skilled workers are attracted by places where firms are many (see (5)). This agglomeration force is called the *price index effect*.

[5] Doing so reduces the proliferation of parameters in the model. See Neary (2001) and Baldwin et al. (2003) for a discussion.

[6] Note that (4) implies a multiplicative markup pricing since the marginal cost is equal to $\beta w_r < w_r$ because our normalisation implies $\beta < 1$.

Using (4)–(6), we find that the typical firm established in region r breaks even if

$$w_r = w_r^{1-\sigma}\frac{\mu Y_r}{n_r w_r^{1-\sigma} + \tau^{1-\sigma} n_s w_s^{1-\sigma}} + \tau^{1-\sigma} w_r^{1-\sigma}\frac{\mu Y_s}{\tau^{1-\sigma} n_r w_r^{1-\sigma} + n_s w_s^{1-\sigma}}.$$

Obviously, this expression is nonlinear in w_r, which explains why most of the economic geography literature relies on simulations for most of their results. As we shall see, we can nevertheless characterize most interesting aspects of the model with pencil and paper.

We also need an expression for the unskilled labor as well as an expression for the agricultural good markets clearing conditions. By our assumptions and normalizations about the agricultural sector, these are the same. In particular, they are given by

$$L = (1-\mu)(w_1 H_1 + w_2 H_2 + L)$$

Inter-regional trade balances regional excesses in supply and demand across sectors. The skilled labor supply in regions 1 and 2 is given by λH and $(1-\lambda)H$, respectively. Thus, the foregoing expression is equivalent to

$$\frac{[\lambda w_1 + (1-\lambda)w_2]H}{L} = \frac{\mu}{1-\mu} \tag{9}$$

which states that the ratio of the wage bills of the two types of labor is equal to the corresponding ratio of the two expenditure shares.

Finally, substituting (7) in (3) yields the skilled labor demand from a typical firm located in region r. Hence, given the normalization above, each skilled labor market clearing implies that the number of firms located in regions 1 and 2 as well as the total mass of firms, are such that:

$$n_1 = \lambda H \quad n_2 = (1-\lambda)H \quad n = n_1 + n_2 = H$$

Thus, the total number of firms (n) is constant but their spatial distribution is endogenous because skilled workers are mobile. In addition, there is a one-to-one relationship between workers and firms established in the same region which, therefore, move together.

Firms are attracted by the region where workers are numerous because demand for the manufactured good is high (see (6)). This agglomeration force is called the *market size effect*, which strengthens the price index effect to generate a circular causation, driving the agglomeration of firms and skilled workers into one region. By contrast, the immobility of the unskilled is a dispersion force.

A spatial equilibrium is the outcome of the interplay between those agglomeration and dispersion forces and is such that no skilled worker has an incentive to change location. It depends on the transport cost level (Krugman, 1991). Formally, denoting by $V_i(\lambda)$ the indirect utility a skilled worker enjoys in region i=1, 2, a *spatial equilibrium* arises at $\lambda \in (0, 1)$ when

$$\Delta V(\lambda) \equiv V_1(\lambda) - V_2(\lambda) = 0$$

or at λ=0 when $\Delta V(0) \leq 0$, or at λ=1 when $\Delta V(1) \geq 0$. Such an equilibrium always exists because $V_r(\lambda)$ is a continuous function of λ (Ginsburgh et al., 1985). A spatial equilibrium is (locally) *stable* if, for any marginal deviation of the population distribution from the equilibrium, the equation of motion

$$\dot{\lambda} = \lambda(1-\lambda)\Delta V(\lambda)$$

brings the distribution of skilled workers back to the original one.

2.2. Properties of the spatial equilibrium

It is well known that the model presented in the foregoing section leads to very contrasted spatial equilibria according to the level of transport costs. We summarize below the main conclusions derived so far from the core–periphery model; details of the proofs may be found in Fujita et al. (1999), Fujita and Thisse (2002), Neary (2001) and Baldwin et al. (2003).[7] Agglomeration, namely λ=0, 1, is the unique stable equilibrium, whatever the transport cost level, when varieties are very differentiated, that is, when $\sigma < \bar{\sigma}$ (which is called the black hole condition) with

$$\bar{\sigma} \equiv \frac{1}{1-\mu} \tag{10}$$

When the no black hole condition holds ($\sigma \geq \bar{\sigma}$), agglomeration is a stable equilibrium if $\tau \in [1, \tau_s]$ where τ_s, called the *sustain point*, is the unique solution in the interval $(1, \infty)$ of the equation:

$$\frac{1-\mu}{2}\tau^{\sigma(1-\mu)-1} + \frac{1+g\mu}{2}\tau^{1-\sigma(1+\mu)} = 1$$

whereas dispersion, namely λ=1/2, is a stable equilibrium, if transport costs are above the threshold value τ_b, called the *break point*, defined as

$$\tau_b = \left\{ \frac{(1+\mu)[\sigma(1+\mu)-1]}{(1-\mu)[\sigma(1-\mu)-1]} \right\}^{1/(\sigma-1)}$$

Furthermore, we have

$$\tau_s > \tau_b$$

so that both dispersion and agglomeration are stable equilibria when transport costs lie within the interval $[\tau_b, \tau_s]$. When τ lies in the interval (τ_b, τ_s), there also exist two interior, asymmetric equilibria, symmetric around λ=1/2, but these are unstable.

[7] A comprehensive study of the set of equilibria and of their stability properties in this and related models can be found in Robert-Nicoud (2005).

Let us now show that, at any stable spatial equilibrium, prices as well as nominal wages are equal and constant for all skilled workers regardless of the type of equilibrium—agglomeration (A) or dispersion (D). When the economy involves dispersion, we have λ=1/2 and w_1=w_2≡w^{D}, where (9) implies that

$$w^{\mathrm{D}} = \frac{\mu}{1-\mu}\frac{L}{H}$$

When the economy involves agglomeration (in region 1, say), we have λ=1 and w_1≡w^{A}, where (9) leads to

$$w^{\mathrm{A}} = \frac{\mu}{1-\mu}\frac{L}{H} = w^{\mathrm{D}}$$

Hence, we have shown:

Lemma 1. *Nominal wages of the skilled workers are identical, whether the economy is dispersed or agglomerated.*

So far, we have implicitly identified a worker of a given type with a labor unit of that type. In what follows, we use two additional normalizations in order to simplify the expression of w^{D} and w^{A}. More precisely, we choose the unit of skilled labor and the unit of unskilled labor in the economy (but *not* the number of workers) to satisfy the equalities H=μ and L=1−μ, from which it follows immediately that

$$w_r = w^{\mathrm{D}} = w^{\mathrm{A}} = 1 \tag{11}$$

Since w_a=1, we have:

Lemma 2. *If the unit of skilled (resp., unskilled) labor is chosen for the proportion of skilled (resp., unskilled) workers to be equal to the share of expenditure on the manufactured (resp., agricultural) good, whatever the market equilibrium, the nominal wages per unit of skilled and unskilled labor are identical for both types of workers.*

Hence, per unit labor real wages vary between agglomeration and dispersion only with the price index.[8] Using (4) then shows that the price of local varieties is the same (and equal to 1) in both regions at either equilibrium. Consequently, (11) implies that *indirect utility (and thus welfare)varies only with the spatial distribution of firms through the price index*. Indeed, substituting (5) into (8), we obtain:

$$V_r = A\left(n_r + \tau^{1-\sigma} n_s\right)^{\mu/(\sigma-1)} \tag{12}$$

Note that this expression is valid only for $n_r \in \{0, H/2, H\}$, for otherwise w_r and p_r are different from unity.

[8] Note that Lemma 2 does not mean that an unskilled worker earns as much as a skilled worker because the number of skilled labor units embodied in the former need not be equal to the number of unskilled labor units embodied in the latter.

When there is dispersion, let q_{rr}^{D} (resp., q_{sr}^{D}) be the equilibrium consumption of a worker located in region r of any variety produced in region r (resp., region s). The expressions of q_{sr}^{D} and q_{rr}^{D} are respectively given by

$$q_{rr}^{\mathrm{D}} = \mu P_r^{\sigma-1} = \frac{2}{1+\tau^{1-\sigma}} \quad q_{sr}^{\mathrm{D}} = \mu P_r^{\sigma-1}\tau^{-\sigma} = \frac{2\tau^{-\sigma}}{1+\tau^{1-\sigma}}$$

thus implying that $q_{rr}^{\mathrm{D}} = \tau^{\sigma} q_{sr}^{\mathrm{D}}$. Note that, when dispersion prevails, the consumption of a local (or of an imported) variety is the same whatever the location and the type of worker.

When there is agglomeration, the equilibrium consumptions of a worker located in region 1 (the core) and in region 2 (the periphery) are the same within each region but differ across regions. They are now given by:

$$q_{11}^{\mathrm{A}} = \mu P_1^{\sigma-1} = 1 \quad q_{12}^{\mathrm{A}} = \tau^{-1} q_{11}^{\mathrm{A}} = \tau^{-1}.$$

Since $\tau>1$ and $\sigma>1$, we have the following ranking of those various consumption levels:

$$q_{rr}^{\mathrm{D}} > q_{11}^{\mathrm{A}} > q_{12}^{\mathrm{A}} > q_{sr}^{\mathrm{D}}$$

The following comments are in order. First, under agglomeration, because of transport costs, workers living in the core consume more of each variety than those living in the periphery ($q_{11}^{\mathrm{A}} > q_{12}^{\mathrm{A}}$). Second, under dispersion, the presence of transport costs leads workers to consume more of each of the locally produced varieties and less of each of those produced abroad ($q_{rr}^{\mathrm{D}} > q_{sr}^{\mathrm{D}}$). The last two comparisons are less straightforward. Because of transport costs, workers substitute local varieties to imported varieties and end up consuming more of each of the locally produced varieties under dispersion than under agglomeration in which they equally consume each variety ($q_{rr}^{\mathrm{D}} > q_{11}^{\mathrm{A}}$). Similarly, although no variety is locally produced in the periphery, workers living in the periphery consume more of each imported variety under agglomeration than they do under dispersion ($q_{12}^{\mathrm{A}} > q_{sr}^{\mathrm{D}}$), because there is no substitution effect within the range of varieties.

We now have everything at hand to address the core issue of this paper.

3. Welfare analysis of the market equilibrium

3.1. Does agglomeration Pareto dominate dispersion, or vice versa?

The welfare of a skilled worker when there is agglomeration in region 1 is denoted V_1^{A}. When there is dispersion the welfare level is the same for all workers, $V_1^{\mathrm{D}} = V_2^{\mathrm{D}} = V^{\mathrm{D}}$, because the number of firms located in each region is the same ($n_1 = n_2 = \mu/2$). For convenience, we set $A \equiv \mu^{-\mu/(\sigma-1)}$. Using (12), it is readily verified that V_1^{A} and V^{D} are as follows:

$$V_1^{\mathrm{A}} = 1 \quad V^{\mathrm{D}} = \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu}{\sigma-1}} \tag{13}$$

We now determine when $V_1^A > V^D$. This holds true if and only if $1 > [(1+\tau^{1-\sigma})/2]^{\frac{\mu}{\sigma-1}}$, that is, if and only if $1 > \tau^{1-\sigma}$, which is always satisfied because $\tau > 1$ (and $\sigma > 1$). Thus, we have the following result:

Proposition 3. *Whatever the level of transport costs, all skilled workers prefer agglomeration to dispersion.*

Consider the welfare of the unskilled in each equilibrium. The welfare of an unskilled worker located in the core is the same as the one of a skilled worker. Therefore, her well-being increases when agglomeration arises. By contrast, an unskilled worker living in the periphery is worse off. Indeed, such a worker prefers agglomeration to dispersion when $V_2^A \geq V^D$ with:

$$V_2^A = \tau^{-\mu} \tag{14}$$

Using (13), we can see that $V_2^A \geq V^D$ if and only if $\tau^{-\mu}[(1+\tau^{1-\sigma})/2]^{\frac{\mu}{\sigma-1}}$, that is, $\tau < 1$, which is impossible. Thus, we have:

Proposition 4. *Whatever the level of transport costs, all the unskilled located in the periphery prefer dispersion to agglomeration whereas all the unskilled located in the core prefer agglomeration to dispersion.*

It follows from Propositions 3 and 4 that *neither agglomeration nor dispersion is a Pareto dominant allocation*: $V_1^A > V^D > V_2^A$. This result suffices to show that there is a conflict of interest between the two geographical groups of unskilled. The agglomeration of firms and skilled workers in one region implies a fall in the price index in this region, but leads to an increase in the price index in the other region because the unskilled living there now have to bear the transport costs of all varieties of the manufactured good. Hence, when skilled workers move from one region to the other, they impose a positive external effect on the immobile workers located in the core but a negative external effect on the immobile workers located in the periphery.

3.2. Agglomeration, dispersion, and potential Pareto improvements

Without carrying out interpersonal comparisons, a compensation criterion based on the prevailing equilibrium prices and wages may be used in order to determine whether the level of social welfare rises when agglomeration or dispersion prevails. Restricting ourselves to these two configurations is legitimate because they are the only market equilibria. Let us describe how the compensation mechanism works. Without loss of generality, we can compute the compensation schemes in terms of labor units. Indeed, as preferences are homothetic, the compensation paid or received by an individual worker is equal to the compensation associated with her type of labor times the number of labor units she owns.

Consider the following two states for the economy: agglomeration (A) and dispersion (D). Assume that agglomeration prevails, so that the workers living in the core (resp., periphery) are better off (resp., worse off) than under dispersion. We then ask the question: is A preferred to D? Two cases must then be distinguished. In the former, we follow Kaldor (1939) and say that A is preferred to D when the winners are able to compensate the losers in order to give them the utility level they would reach under dispersion. In the latter, we follow Hicks (1940) and say that A is preferred to D when the losers are not able to compensate the winners by giving them the utility level they would reach under agglomeration, when the economy moves to dispersion. As argued by Scitovsky (1941), both criteria must be satisfied for A to be preferred to D (or vice versa).

In either case, we take as given the equilibrium prices of the varieties as well as the equilibrium wages to determine the compensations to be paid. Consequently, for the compensations to be feasible, it must be that the individual consumptions of each variety evaluated at the *incomes net of compensation* add up to the quantity supplied by each firm. In this case, the material balance conditions hold true so that the equilibrium prices remain the same. These conditions must also be satisfied for the firms to be able to pay the equilibrium wages on which the compensations are based (see (11)).

Consider now the above two criteria.

1. *When does agglomeration (λ=1) correspond to a potential Pareto improvement compared to dispersion (λ=1/2)?* More precisely, when agglomeration prevails, we must determine if an appropriate redistribution of income from those workers residing in the core – who are better off – can keep unchanged the utility level of the unskilled residing in periphery – who are worse off.

The argument involves three steps, all evaluated at the corresponding equilibrium prices and wages: (i) we compute the transfers needed to compensate those who are in the periphery, (ii) we then check that the material balance conditions still hold after compensation, (iii) finally, we determine under which condition (if any) the level of welfare net of transfers of each (skilled or unskilled) worker living in the core must exceed the well-being she would get under dispersion without compensation.[9]

(i) For the unskilled in the periphery to be exactly compensated, they must be given an additional income $C_{\rm A}$ such that their utility level at the agglomerated configuration (see (14)) after compensation equals their utility level $V^{\rm D}$ prevailing at the dispersed configuration (see (13)), that is,

$$\tau^{-\mu}(1+C_{\rm A}) = \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu}{\sigma-1}}$$

whose solution is

$$C_{\rm A} = \left(\frac{1+\tau^{1-\sigma}}{2\tau^{1-\sigma}}\right)^{\frac{\mu}{\sigma-1}} - 1 \tag{15}$$

which is positive because τ>1.

[9] From (11), it is clear that prices and wages are the same at any (stable) spatial equilibrium. However, prices and wages differ out of these equilibria. As a consequence, the compensation under consideration cannot be considered as non-distortionary in general.

Since the residents in the periphery have the same welfare level and face the same prices and wage, the total compensation paid by those living in the core must be equal to

$$\frac{1-\mu}{2}C_{\mathrm{A}}$$

Since the residents in the core have the same welfare level and face the same prices and wage, each resident in the core must then pay the same amount $T_{\mathrm{A}}>0$ given by

$$T_{\mathrm{A}} = \frac{1-\mu}{1+\mu}C_{\mathrm{A}} \tag{16}$$

which decreases as the size of the manufactured sector rises. After compensation, the income of a worker in the core is $1-T_{\mathrm{A}}$.

(ii) We must now determine if the material balance conditions still hold at the consumption pattern corresponding to the compensated incomes given by

$$Y_1 = \frac{1+\mu}{2}(1-T_{\mathrm{A}}) \quad Y_2 = \frac{1-\mu}{2}(1+C_{\mathrm{A}})$$

At the agglomerated configuration, the total consumption of every variety *after* compensation is given by the sum of the consumption by each of the workers who are in the core augmented by the consumption of each of the unskilled who are in the periphery. It follows from (4), (6), and (11) that the total demand for any variety i – which are here all produced in region 1 – is such that

$$D_1(i) = \mu\left(P_1^{\sigma-1}Y_1 + \tau^{1-\sigma}P_2^{\sigma-1}Y_2\right)$$

where, given (5)

$$P_1 = \mu^{\frac{1}{1-\sigma}} \quad P_2 = \tau\mu^{\frac{1}{1-\sigma}}$$

Replacing in $D_1(i)$ yields

$$D_1(i) = \mu\left[\frac{1}{\mu}\frac{1+\mu}{2}(1-T_{\mathrm{A}}) + \tau^{1-\sigma}\frac{\tau^{\sigma-1}}{\mu}\frac{1-\mu}{2}(1+C_{\mathrm{A}})\right] = 1$$

with the second equality stemming from (15) and (16); this is just equal to the equilibrium production of a firm (see (7)). This production level allows each of them to pay the equilibrium wage (w_r=1) used to calculate the compensation.

It follows from the Walras law that the market clearing condition for the agricultural good holds.

(iii) Finally, every worker in the core strictly prefers the agglomeration outcome if and only if her welfare level after compensation $1-T_{\mathrm{A}}$ exceeds the welfare level V^{D} she would get under dispersion, namely

$$1-T_{\mathrm{A}} > \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu}{\sigma-1}}$$

Using (15) and (16), this is equivalent to

$$F(\tau) \equiv \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{-\mu}{\sigma-1}} - \left(\frac{1+\mu}{2} + \frac{1-\mu}{2}\tau^{\mu}\right) > 0. \tag{17}$$

As shown in Appendix A, there exists a single value of $\tau > 1$, denoted τ_K, such that $F(\tau) = 0$ and $F(\tau) > 0$ if and only if $\tau < \tau_K$.

Consequently, when the economy moves from dispersion to agglomeration, *A is preferred to D in the sense of Kaldor as long as $\tau < \tau_K$ because the welfare benefits earned by those living in the core are sufficiently large to compensate those who stay in the periphery.* By contrast, when $\tau \geq \tau_K$, D is preferred to A.

2. *When does dispersion ($\lambda = 1/2$) correspond to a potential Pareto improvement compared to agglomeration ($\lambda = 1$)?* In other words, we want to know if, under dispersion, an appropriate redistribution of income from the unskilled who would otherwise be living in the periphery can keep unchanged the utility level of those who would live in the core, without making those unskilled worse off.

The answer is here straightforward. The payment of any compensation makes the spatial distribution of income uneven between regions. This in turn prevents the wages and prices corresponding to the dispersed configuration to balance the product and labor markets. Consequently, when the economy moves from dispersion to agglomeration, *the unskilled in region 2 are unable to compensate the other workers at the prevailing market prices and wages for the economy to move (back) toward dispersion*. Hence, A is always preferred to D in the sense of Hicks.

Accordingly, when $\tau < \tau_K$ agglomeration is preferred to dispersion according to both criteria. By contrast, when $\tau \geq \tau_K$, we are in a situation of *indetermination* in the sense of Scitovsky (1941), which means that no state of the economy is preferred under the two compensation criteria. We may then conclude as follows:

Proposition 5. *When transport costs are sufficiently low, agglomeration is socially preferable to dispersion in the sense of both Kaldor's and Hicks' criteria. Otherwise, it is impossible to discriminate between these two configurations by using those criteria.*

In order to assess the practical relevance of this result, we compute τ_K for different values of σ and μ chosen according to the following criteria. When the industrial sector stands for all tradable goods in the economy, estimations of σ vary between 3 and 5 whereas μ takes values between 0.5 and 0.8 (Head and Mayer, 2004). By contrast, when this sector is a specific industry, σ sharply rises because varieties are now much better substitutes than in the aggregate level; a value of σ close to 7 is then acceptable. In this case, μ typically takes a value lower than 0.2, which corresponds approximately to the share of the manufactured good in a developed economy. Table 1, where the value between parentheses is the corresponding break point, reveals several interesting things.

First, the threshold τ_K seems to be lower than the break point τ_b so that the market would yield agglomeration for values of the transport costs that exceed the threshold below which it is socially desirable. When there is a black hole, i.e. $\sigma \leq 1/(1-\mu)$, agglomeration is always the market equilibrium, which means that $\tau_b \to \infty$ and $\tau_s \to \infty$. Because τ_K is strictly larger than 1 and takes a finite value, it must be that $\tau_K < \tau_b$, τ_s. By

Table 1
Critical values of τ for the compensation mechanisms

	μ=0.2	μ=0.4	μ=0.6
σ=3	1.45 (1.67)	2.07 (3.05)	3.11 (8.72)
σ=5	1.22 (1.26)	1.49 (1.63)	1.94 (2.30)
σ=7	1.14 (1.16)	1.32 (1.36)	1.60 (1.68)

continuity, we get $\tau_K<\tau_b$ for values of σ and μ that are just away from the domain defined by the black hole condition. However, when we move to parameter configurations that are sufficiently far from this domain whence agglomeration forces are weak, we obtain $\tau_b<\tau_K$. For example, for μ=0.6, we have $\tau_b<\tau_K$ as long as σ exceeds 12. Consequently,

Proposition 6. *No general conclusion may be drawn in the Krugman core–periphery model as to whether the market yields excessive agglomeration.*

Yet, using plausible values of the parameters μ and σ as well as the value of the transport cost τ estimated by Head and Ries (2001) from Canada–US trade data (after having controlled for other spatial differences), *excessive agglomeration seems to occur*. This is worth noting because the North American space is likely to be more integrated than the European Union (Head and Mayer, 2004).

Second, for relatively low values of μ, the threshold τ_K is lower than the estimate of the transport cost τ obtained by Head and Ries (2001), thus suggesting that τ_K takes values that are plausible. Last, the gap between τ_b and τ_K sharply rises when the economy moves from the lower left corner of the table toward the upper right corner, namely when we consider a larger industrial sector producing more differentiated varieties. In this case, the market outcome seems to be inefficient over an expanding range of transport cost values.

4. Market equilibrium and social welfare functions

The partial indetermination established in Proposition 5 leads us to retain particular, but meaningful, social welfare functions (SWF), which are often used in public economics. Although the SWF approach is debatable (Fleurbaey and Hammond, 2004), we believe that it contributes to fill up the gap (at least partially) in the welfare analysis of the core–periphery model.

In the case of n agents whose utility is $u_i(s)$ when the state of the economy is s, we focus on the following class of symmetric CES-type SWF:

$$W(s) = \begin{cases} \frac{1}{1-\eta}\sum_{i=1}^{n}[u_i(s)]^{1-\eta} & \text{for } \eta\neq 1 \\ \sum_{i=1}^{n}\ln u_i(s) & \text{for } \eta = 1 \end{cases} \tag{18}$$

in which $\eta\geq 0$ measures the degree of aversion toward inequality. In particular, when η=0 (or zero aversion to inequality) the function W is identical to the *utilitarian* welfare function in which the sum of all workers' (indirect) utilities is maximized. At the other extreme, when $\eta \rightarrow \infty$ (or infinite aversion to inequality), we have the *Rawlsian* welfare function, which only values the (indirect) utility of the worst-off worker. Intermediate values of η express different societal attitudes toward economic inequality among individuals, that is, among three groups of workers in our setting: as η rises from zero to infinity, the bias in favor of the disadvantaged increases.

As in the foregoing, we want to compare two states of the economy: agglomeration and dispersion. However, *we must now account for the fact that we sum over individual workers and not over units of labor*. To this end, we denote respectively by S and U the mass of skilled and unskilled workers. All skilled (resp., unskilled) workers supply the same amount of skilled (resp., unskilled) labor. Since the total wage bill of the skilled is equal to μ, the earnings of each skilled worker are equal to μ/S. Similarly, the nominal income of an unskilled worker is given by $(1-\mu)/U$.

As individual preferences are homothetic, in the case of agglomeration our SWF may then be written as follows:

$$W(A) = \frac{1}{1-\eta}\left[S\left(\frac{\mu}{S}V_1^{\mathrm{A}}\right)^{1-\eta} + \frac{U}{2}\left(\frac{1-\mu}{U}V_1^{\mathrm{A}}\right)^{1-\eta} + \frac{U}{2}\left(\frac{1-\mu}{U}V_2^{\mathrm{A}}\right)^{1-\eta}\right]$$

where V_1^{A} and V_2^{A} are given in (13) and (14). When dispersion prevails, we have:

$$W(D) = \frac{1}{1-\eta}\left[S\left(\frac{\mu}{S}V^{\mathrm{D}}\right)^{1-\eta} + U\left(\frac{1-\mu}{U}V^{\mathrm{D}}\right)^{1-\eta}\right]$$

where V^{D} is given by (13). It is then easy to see that

$$G(\tau;\eta) \equiv W(A) - W(D) = \left(\frac{1-\mu}{U}\right)^{1-\eta}\Delta(\tau;\eta)$$

where

$$\Delta(\tau;\eta) \equiv \frac{1}{1-\eta}\left[\left(S\omega^{1-\eta} + \frac{U}{2} + \frac{U}{2}\tau^{-\mu(1-\eta)}\right) - \left(S\omega^{1-\eta} + U\right)\left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu(1-\eta)}{\sigma-1}}\right]$$

and the relative wage of a skilled to an unskilled ω is equal to

$$\omega = \frac{\mu}{1-\mu}\frac{U}{S}$$

by (9).

Let

$$\tau' \equiv \left[2\left(\frac{U}{S\omega^{1-\eta} + U}\right)^{\frac{-(\sigma-1)}{\sigma-1-\mu(1-\eta)}} - 1\right]^{1/(\sigma-1)}$$

be the value of transport costs such that $d\Delta/d\tau=0$, and

$$\hat{\sigma}(\eta)\equiv 1+(1-\eta)\frac{\mu\log 2}{\log(S\omega^{1-\eta}+U)-\log(S\omega^{1-\eta}+U/2)}$$

the elasticity of substitution for which $\lim_{\tau\to\infty}\Delta(\tau;\eta)>0$. These two specific thresholds lead to the following result, proven in Appendix B.

Proposition 7. *When varieties are very differentiated ($\sigma\leq\hat{\sigma}(\eta)$), agglomeration is socially preferred to dispersion, whatever the level of transport costs. When they are not ($\sigma>\hat{\sigma}(\eta)$), there exists a unique transport cost value $\tau^*(\eta)>1$ such that agglomeration (resp., dispersion) is socially preferred to dispersion (resp., agglomeration) when $\tau<\tau^*(\eta)$ (resp. $\tau>\tau^*(\eta)$).*

In words, when varieties are very differentiated ($\sigma<\hat{\sigma}(\eta)$), agglomeration is socially preferred to dispersion, even when transport costs are high. Indeed, as varieties are poor substitutes, workers have a strong preference for variety, thus implying that the consumption of imported varieties is just slightly below that of local varieties. Agglomeration is then more desirable than dispersion because less individuals consume imported varieties in agglomeration than in dispersion, thus implying that the quantities of shipped varieties are lower. Note, however, that η must be lower than 1 for the threshold $\hat{\sigma}(\eta)$ to be larger than 1. Hence, for agglomeration to be always preferred to dispersion, the degree of aversion toward inequality must be low. By contrast, when $\sigma\geq\hat{\sigma}(\eta)$, varieties are more substitutable, so that dispersion is more desirable than agglomeration for sufficiently large values of transport costs because consumers are now biased toward local varieties.

So far, we have not investigated the connections between the SWF approach and the market outcome. We show below how this can be accomplished by focusing on particular values of η. To start with, we consider the case where there is no aversion toward inequality ($\eta=0$). Proposition 7 shows that agglomeration is always desirable from the utilitarian standpoint when $\sigma<\hat{\sigma}(0)$. Therefore, as $\hat{\sigma}(0)$ can be shown to be larger than $\bar{\sigma}$, the market outcome is always preferable from the utilitarian standpoint when the black hole condition prevails. Yet, we have seen that $\tau<\tau_K$ must hold for agglomeration to be efficient in Kaldorian terms. Thus, because it does not account for inequality across individuals, utilitarianism is biased toward agglomeration.

Given the role played by the utilitarian criterion in public economics, it is interesting to assess the performance of the market outcome against it when the black hole condition does not hold. To this end, we must rank the thresholds $\tau^*(0)$, τ_b and τ_s (where τ_b and τ_s are defined in Section 2).[10] Recall that these three values depend only upon μ and σ. For $\sigma=\hat{\sigma}(0)$, we have $\Delta(\tau;\ 0)\to 0^-$ when $\tau\to\infty$. In this case, it is readily verified that $\tau_b<\tau_s<\tau^*(0)\to\infty$ for all admissible values of μ. By continuity, the same inequalities remain valid when σ slightly exceeds $\hat{\sigma}$. We know that both τ_b and τ_s decrease when σ rises (Fujita et al., 1999, p. 75), whereas $d\tau^*(0)/d\sigma<0$ so that all thresholds moves to the left with an increase of the elasticity of substitution. However, simulations reveal that $\tau^*(0)$ moves faster than τ_s once σ takes sufficiently large values. The equations being

[10] Observe that $\tau^*(0)$ does not depend on the mass of unskilled and skilled workers.

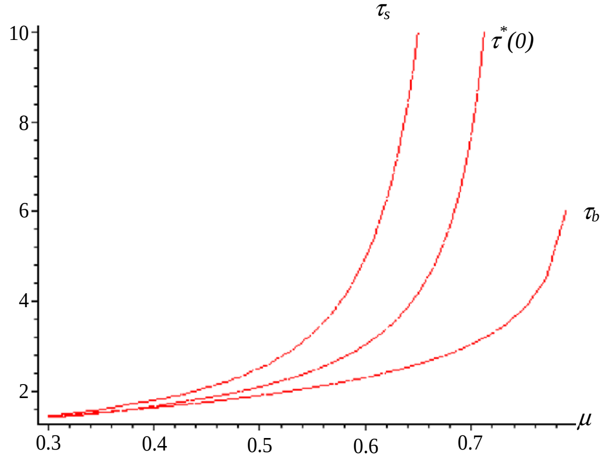

Fig. 1. (σ=5).

nonlinear, we find it very hard to provide an analytical derivation of these properties. Instead, we present some numerical illustrations. Fig. 1 plots $\tau^*(0)$, τ_b and τ_s against μ for σ=5, while Fig. 2 plots these values against σ for μ=0.7. On both figures, we see that $\tau_b<\tau^*(0)<\tau_s$. This suggests the following results: *the market outcome is desirable from the utilitarian standpoint as long as* $\tau>\tau_s$ *or* $\tau<\tau_b$. When $\tau_b<\tau<\tau_s$, both agglomeration and dispersion are equilibria, but dispersion (resp., agglomeration) is socially desirable if $\tau^*(0)<\tau<\tau_s$ (resp., $\tau_b<\tau<\tau^*(0)$).

Note also that for sufficiently large degrees of aversion to inequality (η>1), there always exists a threshold on τ such that dispersion is socially preferred to agglomeration for transport cost values exceeding that threshold. By the same token, for any finite degree of aversion to inequality, the proof in Appendix B shows that there is always a

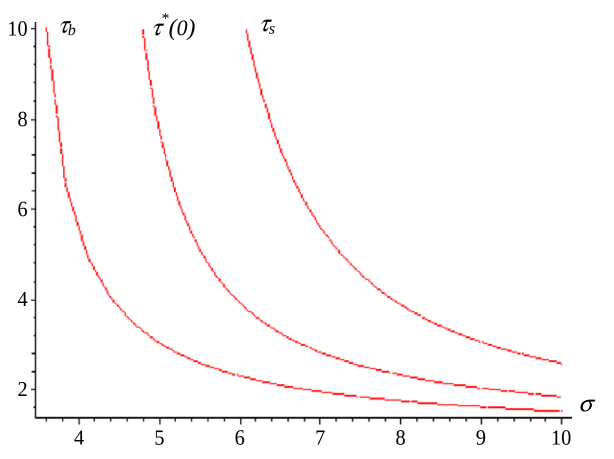

Fig. 2. (μ=0.7).

nonempty range of transport costs values for which agglomeration is preferable. *It is only in the extreme case of a Rawlsian SWF that dispersion is always preferred to agglomeration*. Hence, the market outcome is desirable from the Rawlsian standpoint as long as τ exceeds the sustain point τ_s, but yields the opposite outcome once τ is less than the break point τ_b.

5. Concluding remarks

Using Krugman's core–periphery model, we have investigated whether there are conditions under which agglomeration is socially better than dispersion, or vice versa. Our results show that there is no simple answer to this question. In particular, our indetermination result might explain a posteriori the reticence of some scholars to perform welfare analyses in economic geography models. However, two main policy conclusions emerge from this paper: (i) interregional transfers are likely to be the only strategy allowing one to prevent economic integration-led agglomeration to hurt those living in the periphery and (ii) when such transfers are not implemented, our analysis points to the likely existence of a trade-off between efficiency and equity across individuals.

First, we find it hard to recommend a move from a stable equilibrium, such as agglomeration, to a socially preferred unstable equilibrium, such as dispersion. Even when both equilibria are stable, it seems very problematic for the government to undertake a large scale reorganization of the spatial pattern of activities, because controlling the mobility of agents is politically unfeasible.[11] Instead, we have seen that those who benefit from agglomeration often have the potential for compensating the workers living in the periphery because agglomeration creates rents that can be taxed without the mobile factor leaving the core (Baldwin et al., 2003, chapter 15). This suggests that *interregional transfers could well be the solution for correcting the discrepancies arising in a core–periphery structure*. It is worth stressing that such transfers do not rest here on equity considerations, but on efficiency grounds only. However, implementing such transfers paid for by the workers in the core may be politically difficult: the majority of workers is located in the core against a minority in the periphery. Evidence related to such political economy effects may be found in several European countries such as Belgium, Italy, and Spain.[12]

Second, our results invite us to be careful about recommendations based on the maximization of a utilitarian welfare function when considered as a measure of efficiency. Furthermore, the use of social welfare functions exhibiting different attitudes toward inequality reveals that the evaluation of the market outcome heavily depends on societal values. In particular, our analysis reveals the existence of a trade-off between efficiency and individual equity. This is because the skilled want to get together within what becomes

[11] In the same spirit, note that empirical evidence casts doubt on the ability of real economies to switch away from agglomeration (Davis and Weinstein, 2002).

[12] See Robert-Nicoud and Sbergami (2004) for a possible theoretical justification of these effects.

the core that regional imbalances arise. As the periphery is formed by less mobile and unskilled workers, regional imbalances amount here to the existence of an unequal treatment of a priori identical individuals (the unskilled) based on the behavior of another group of individuals (the skilled). There lies a clear case for redistribution.

As a last comment, note that, although we have followed a different research strategy, our welfare analysis points to the same direction as Ottaviano and Thisse (2002). This confirms the idea that both settings (Krugman, 1991; Ottaviano et al., 2002) belong to a broader class of economic geography models sharing similar properties. The study of this class of models should be given more attention in future research.

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Appendix A

Let

$$F(\tau) \equiv \left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{-\mu}{\sigma-1}} - \left(\frac{1+\mu}{2} + \frac{1-\mu}{2}\tau^{\mu}\right)$$

and set

$$\phi \equiv \tau^{1-\sigma}$$

Then, $F(\tau)$ may be rewritten as follows:

$$f(\phi) \equiv \left(\frac{1+\phi}{2}\right)^{\frac{\mu}{1-\sigma}} - \left(\frac{1+\mu}{2} + \frac{1-\mu}{2}\phi^{\frac{\mu}{1-\sigma}}\right)$$

which is defined on [0, 1]. Note first that $f(1)=0$ whereas $f(\phi) \rightarrow -\infty$ when $\phi \rightarrow 0$. It is readily verified that $f(\phi)$ has a single extremum at

$$\phi^* = \frac{1}{2(1-\mu)^{-\frac{\sigma-1}{\sigma-1+\mu}} - 1}$$

which can be shown to belong to (0, 1). Because

$$\lim_{\phi 1} f'(\phi) = \frac{\mu^2}{2(1-\sigma)} < 0$$

$f(\phi)$ is decreasing in the left neighborhood of $\phi=1$. This in turn implies that, as ϕ varies from 1 to 0, $f(\phi)$ first increases, reaches its maximum at ϕ^* and, finally, decreases without bound. As a result, $f(\phi)=0$ for a single value $\phi_K<1$ such that $f(\phi)>0$ if and only if $\phi>\phi_K$. Let τ_K the value of τ which corresponds to ϕ_K. Hence, $F(\tau)>0$ if and only if $\tau<\tau_K$.

Appendix B

Proof of proposition 7. Recall that

$$G(\tau;\eta) = \left(\frac{1-\mu}{U}\right)^{1-\eta}\Delta(\tau;\eta)$$

where

$$\Delta(\tau;\eta) \equiv \frac{1}{1-\eta}\left[\left(S\omega^{1-\eta} + \frac{U}{2} + \frac{U}{2}\tau^{-\mu(1-\eta)}\right) - \left(S\omega^{1-\eta}+U\right)\left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu(1-\eta)}{\sigma-1}}\right].$$

Clearly, we have $G(1;\eta)=\Delta(1;\eta)=0$. To determine the sign of $G(\tau;\eta)$ for $\tau>1$, it is useful to compute its derivative, hence the derivative of $\Delta(\tau;\eta)$, with respect to τ:

$$\frac{\mathrm{d}\Delta}{\mathrm{d}\tau} = \frac{\mu}{2}\left[-U\tau^{-\mu(1-\eta)-1} + \left(S\omega^{1-\eta}+U\right)\tau^{\sigma}\left(\frac{1+\tau^{1-\sigma}}{2}\right)^{\frac{\mu(1-\eta)}{\sigma-1}-1}\right]$$

which is always positive in the neighborhood of $\tau=1$. The equation $\mathrm{d}\Delta/\mathrm{d}\tau=0$ has a single root given by

$$\tau' \equiv \left[2\left(\frac{U}{S\omega^{1-\eta}+U}\right)^{\frac{1-\sigma}{\sigma-[1+\mu(1-\eta)]}} - 1\right]^{1/(\sigma-1)}$$

This root exceeds 1 if and only if

$$\sigma > 1 + \mu(1-\eta)$$

which holds as long as the no black hole condition is satisfied ($\sigma \geq \bar{\sigma}$ where $\bar{\sigma}$ is given by(10)) and $\tau'=1$ when $\eta \to \infty$. Moreover, we have

$$\lim_{\tau\to\infty}\Delta(\tau;\eta) = S\omega^{1-\eta} + \frac{U}{2} - \left(S\omega^{1-\eta}+U\right)\left(\frac{1}{2}\right)^{\frac{\mu(1-\eta)}{\sigma-1}}$$

This limit is positive if and only if $\sigma<\hat{\sigma}(\eta)$, where $\hat{\sigma}(\eta)$ is defined by

$$\hat{\sigma}(\eta) \equiv 1 + (1-\eta)\frac{\mu \log 2}{\log(S\omega^{1-\eta}+U) - \log(S\omega^{1-\eta}+U/2)}$$

Therefore, as long as $\sigma \leq \hat{\sigma}(\eta)$, $G(\tau;\eta)$ is (weakly) positive for all $\tau>1$ (see Fig. 3(a)). It is clear that the threshold $\hat{\sigma}(\eta)$ exceeds 1 if and only if $\eta<1$. As a result, when varieties are sufficiently differentiated for σ to be very small and when the degree of aversion toward inequality is weak enough, agglomeration is always preferred to dispersion.

It remains to consider the case where $\sigma \geq \hat{\sigma}(\eta)$.[13] Then, we know that $\tau'>1$. It is readily verified that $\mathrm{d}G/\mathrm{d}\tau$ is positive for $\tau<\tau'$ and negative for $\tau>\tau'$, regardless of the value of η.

[13] Note that this condition is satisfied for all admissible parameter values if $\eta \geq 1$.

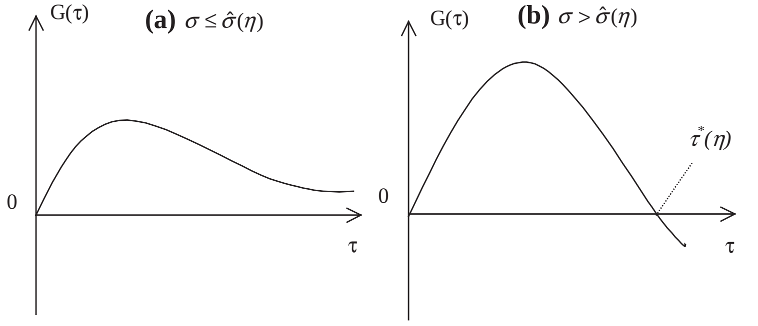

Fig. 3. agglomeration vs. dispersion.

This implies the existence of a unique value $\tau^*(\eta)>\tau'>1$ such that $G[\tau^*(\eta);\eta]=0$ (see Fig. 3(b)). Accordingly, as long as τ is lower than this threshold, agglomeration is preferred to dispersion whereas the opposite holds when τ is larger than $\tau^*(\eta)$.

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